

AN
E P I T O M E
OF
G E O M E T R Y,

BEING

A Compendious Collection of the *First, Third, Fifth, Sixth, Eleventh and Twelfth* Books of *EUCLID*; with their Application to several of the most Useful Parts of the Mathematicks.

A L S O

EUCLID's Second Book and *Doctrine of Proportion*, Algebraically demonstrated.

To which is subjoin'd,

A Treatise of Measuring *Superficies* and *Solids*; *Vulgarly, Decimally and Practically*, with the Customs used by *Artificers* in Measuring their several Works: *Likewise*, Directions for Measuring *Board and Timber*; Making *Vessels* of any Bigness; Taking the Plan of any *Court, Yard, Garden, &c.* Also, The cutting the *Five Regular Bodies*.

Second Edition, caretully Corrected.

By *William Alingham*,
Teacher of the *MATHEMATICKS*.

London, Printed for *Richard and William Mount*, and
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 JANUARY 25, 1924

TO THE
Right Honourable

John, Earl of Bridgewater,
J. Thompson, Lord Haversham,
Sir Robert Rich, Bar.
Sir George Rook, Kt.
Sir David Mitchel, Kt.

COMMISSIONERS, for Executing
the Office of Lord High Admiral of
England.

May it please Your Lordships,

TO Pardon this my Pre-
sumption in Humbly
Presenting this small
piece of *GEOMETRY* to
you, being sensible that none
are more fit to Patronize *Ma-*

The Epistle Dedicatory.

them thematick Sciences than Great Men ; and more especially Your Lordships, to whose Care so weighty a Concern, as the Management of our *Naval Forces* is committed, upon which (next under God and our Gracious King) the sole Welfare of this Nation depends.

I shall not trouble Your Lordships with an Account of the Antiquity and Excellency of *GEOMETRY*, only this I shall say, that no doubt its as old as the World, and that the *Ptolomyes*, who for several Years Sway'd the *Egyptian Scepter*, were great Encouragers

The Epistle Dedicatory.

couragers of this sort of *Learning*, of which the many *Volumns* stow'd in the *Alexandrian Library* (had not the *Flames* destroy'd them) were a sufficient proof; I could also mention divers *Princes* and *Noble Men* of latter Ages, who have been great Lovers of this most Admirable Science.

As to its Utility, daily Experience confirms the great Necessity of it, and particularly and principally *Navigation* and *Marine Architecture*, which without the aid and assistance of this Science would be wholly imperfect and useless.

The Epistle Dedicatory.

Much more might be said
in Commendation hereof, but
I shall desist, and beg a fa-
vourable Censure from your
Lordships of this small, yet (I
am bold to say) useful Trea-
tise, since it has a tendency to
promote the Publick Good,
which if your Lordships shall
please to Vouchsafe, will
thereby Encourage

Your Lordships

Most Humble Servant,

WILLIAM ALINGHAM.

T O T H E
R E A D E R.

TH E great Utility and Excellency of Geometry, not only in several parts of the Mathematicks, but also in humane Affairs is manifest; for as much as upon it are founded most other Sciences, some of which are of such Moment to Man-kind, that without the Knowledge thereof, all our Actions and Transactions would be done in Confusion; So that it needs no Rhetorick to recommend it, the great and continual usefulness, with the no less pleasure that attends its Knowledge, are sufficient Motives, not only to enduce, but Engage one in the Study of it.

For to this Art all Fabricks, from the most magnificent Structure to the meanest Cottage, owe their Contrivance.

Likewise, 'Tis by help of this Science the Country Gentleman may Survey his Lands, Measure his Timber, Contrive his Buildings, Gardens, Fountains, Aquaducts, &c. and if

To the Reader.

he understood it aright, would not repent Exchanging many of his beloved Pastimes, for this greater and more useful Recreation.

The great Delight and Pleasure also of this Study is not less evident to those who are not sworn Enemies to the Art of Thinking, but will allow an Exercise of the Mind join'd with that of the Body, to be the more noble sort of Recreation; For in the consideration of the simplest parts of the Body, viz. Points, Lines and Surfaces, variety of delightful Properties are discover'd; which surprize our Mind with unexpected Truth and Conviction, and lay a Foundation to many Noble and Useful Rules of Practice, not only in several parts of the Mathematicks, but also in many Instances of Common Life.

In short, Time would fail to Enumerate the vast Advantages that attend this most Excellent and almost only Science, to Persons in all Stations, and of all Professions whatsoever, insomuch that no Person can be said to be completely accomplished, without some Competent Knowledge herein.

But

To the Reader.

But no longer to write in praise of Hercules, or commend that which none but inconsiderate Persons did ever discommend, let me come to speak a little of the following Treatise, which I have presum'd to Entitle, *An Epitome of Geometry, or a Compendious Contraction of the First, Third, Fifth, Sixth, Eleventh and Twelfth Books of EUCLID*; in the composing of which, I have carefully endeavour'd to rank the Propositions in such order, as to depend upon each other, and from them have drawn their principal Scholions and Corollaries; Also in the Demonstration hereof, I have not strictly follow'd one particular Method, but have chang'd it to such that I thought most clear and natural to conceive.

To this also is subjoin'd a Treatise of Measuring; in which I have not only given the Geometrical Reason of finding the Content of most of the Bodies herein mentioned, but have also applied them to Practice in the Measuring of Artificers Works, and therein have shown the Customs and Methods now practis'd by them.

In the next place, I have Exhibited the man-

To the Reader.

ner of changing the shape of one Vessel to another and of what Dimensions in any Form, to make them to hold any Quantity of Corn, Liquor, &c.

Lastly, I have shown how to take the Plan of any Court, Yard, Garden, &c. As also the manner of Cutting and Measuring the Five Regular Platonic Bodies.

These are the Principal Matters in short herein Contained, and what I thought meet to acquaint the Reader withal.

What I have else to say is, that if in what is hereafter delivered, any Mistake be committed, or some things not so clearly express'd as they might have been; I desire the Courteous Reader to consider, that the Press and our Natures are the Causes of such Effects; Faults ever attending the former, and Frailty the latter; and though undoubtedly it will meet with the Common Fate of all Books, to wit, Cavillers, yet I hope there are others that will kindly accept of what is here offer'd, and freely Pardon the Errors, remembering that they are Humane.

So leaving the following Treatise to the Reader, wishing him in the perusal hereof, as much Satisfaction as I had in the writing, and then I am sure his End will be obtained, which that it may, is the hearty desire of their Real Friend

W. A.

Notes Explai'd.

$=$	as $a = b$	} is thus Read.	a is Equal to b
$+$	as $a + b$		a more b
$-$	as $a - b$		a less b
$\times$	as $a \times b$		a Multiplied by b
$\sqsupset$	as $a \sqsupset b$		a is greater than b
$\sqsubset$	as $a \sqsubset b$		a is lesser than b
$<$	as $< a$		the angle a
$\triangle$	as $\triangle abc$	}	the Triangle a b c
$\sqrt{}$	as $\sqrt{a}$		the square Root of a
q	as $a q$		the square of a

These four Points ($::$) placed between four Quantities is the Note of Proportion, as if the four Quantities stand in this order, $a : b :: c : d$, they are thus to be read, as a to b so is c to d .

This Note ($\therefore$) before any parcel of Quantities, shews them to be in continual proportion, as $\therefore a, b, c, d, \&c.$ are thus to be read. As a to b , so is c to d .

Two Letters placed in this manner $\overline{a} b$ notes Division, and shews that a must be divided by b .

Also $a \times b + c$ denotes that a is to be multiplied by the Sum of b and c . Understand the like by the difference of two Quantities.

Lastly, $W. W. D.$ stands for *which was to be demonstrated*.

Take Notice also, 1. That in the Schemes of the following Theorems, the Lines drawn for the help of Demonstration are generally speck'd.

2. That in these *Theorems* 'tis sufficient to suppose that Parallel Lines can be drawn, Perpendiculars rais'd, Angles fram'd, and other Figures describ'd; the manner of doing them is shewn in the Problems that come after.

3. That for the Convenience of References, I have put all the *Definitions* together; as also all the *Theorems*, numbering them successively; Notwithstanding which, it will be most proper for the Reader to peruse them severally; that is, those *Definitions* relating to Plain Geometry, before he enter on the Plain *Theorems*; and the other *Definitions*, before those of Solids. And that he may readily know when those *Definitions* begin, I have incerted these words (*Of Solids*) over the first *Definition* and *Theorem*.

G E O M E T R Y.

GEOMETRY, as to its *Name*, signifies properly no more than the *Measuring* of the *Earth*; but in general, 'tis defined to be the *Science* of *Magnitude*, or *Continued Quantity*, whose *Parts*, though never so vast, or remote, are by its own *Principles* and *Demonstrations* understood, and exactly measur'd; so that it hath the whole *Universe* for its *Subjeſt*, and is indeed the moſt pure of all the *Mathematick Sciences*.

It is principally divided into two *Parts*, viz. *Speculative* and *Practick*. *Speculative* is that which is employ'd about demonstrating the *Properties* of *Magnitudes*: The *Practick* is that which teacheth how to describe and measure thoſe *Magnitudes*; and as the firſt hath his *Theorems*, ſo the later hath his *Problems*.

Of this *Science*, I ſhall therefore lay down in the firſt place ſuch *Definitions* and *Principles*, as ought duly to be weigh'd and conſider'd by the *Reader*, before he paſs to the *Demonſtration* and *Practice* of that which follows.

But before we enter on the *Definitions*, it may not be amiſs to inform him of the *kinds* of *Magnitudes*, which are *Three*, viz. *Length*, *Breadth* and *Thickneſs*, or a *Line*, a *Superficie*, and a *Solid*. A *Line* being generated by the motion of a *Point*; a *Superficie* by the motion of a *Line*, and a *Solid* by the motion of a *Superficie*, and which way ſoever a *Solid*

is moved, it still produceth but a *Solid*. Now, what a *Point*, a *Line* and *Superficie* are, the following *Definitions* will declare unto you.

Dimensions.

1. **A** *Point* is that which hath no part, as the prick *A*.

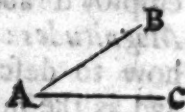
2. A *strait* or *right line*, is the *shortest distance* betwixt two points, as the Line *AB*. Hence two sides of a Triangle taken together, are longer than the third.



3. A *Superficies* is that which hath *length* and *breadth*, but no *thickness*.

4. A *plain Superficies* is the *shortest Space* that can be described between one or more *right Lines*, and is bounded by them.

5. A *plain* or *right lin'd angle* is made by the meeting of two *right lines* *BA*, *CA*, as the Angle *BAC*. Note, the middlemost of the three Letters point out the Angle, as by Angle *BAC* is understood the Angle *A*.



6. A *Circle* is a *plain Figure* enclos'd by one line, call'd a *Periphery*, or *Circumference*, unto which from one Point within the *Figure* all lines drawn are equal, and these lines are called *Radii*, or *Semidiameters*, as the Figure *ABCD*.

7. And that point in the middle of the Figure noted with *E*, is call'd the *Center*.

8. The *Diameter* of a *Circles* is a *strait line* which passeth through the *Center*, and is bounded on both sides by the *Circumference*, dividing the *Circle* into two equal parts, as the line *AC*.



9. A *Semicircle* is half the whole *Circle*, being contain'd betwixt the *Diameter*, and that part of the *Circumference* cut off by it, as *A B C*.

10. A *Segment* of a *Circle*, is a part cut off by a *right line* (less than the *Diameter*) drawn within the *Circle*, and the *Segment* may be either greater or lesser than a *Semicircle*, as the Figure *A* or *Z*.



11. A *Sector* is form'd by part of a *Circle*, and two *Semidiameters* drawn from the *Center* to the *Circumference*, as the Figures *C* and *B*.



12. Every *Circle*, whether great or small, is suppos'd to be divided into 360 equal parts, call'd *Degrees*, and each *Degree* again into 60 *Minutes*, and every *Minute* into 60 *Seconds*, and so to *Thirds*, *Fourths*, &c. The reason of dividing it into 360 parts, is, because this Number will admit of most equal *Divisions*, as 2 3. 5. 6. 7. &c. And therefore the best way to divide a *Circle* as aforesaid, is first to divide a *Circle* as aforesaid, is, first to divide it into 4 equal parts, then one of these into 3 equal parts, next one of these into 6 equal parts, and lastly one of these 6 into 5 equal parts, one of which will be a *Degree*, or the $\frac{1}{360}$ part of the whole *Circle*.

13. The *Measure* of an *Angle*, is an *Arch* of a *Circle* described from the *Angular Point*, and lies intercepted betwixt the sides, as the *Arch B C*. Now so many *Degrees* and *Minutes* as are in the *Arch B C*, so much is the *measure* of it, or of the *Angle A*.

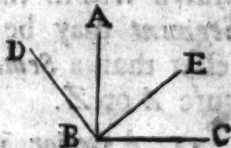


14. A *plain right lin'd Angle* may be *right* or *oblique*.

15. A *Right Angle* is that which hath an *Arch* of 90 Degrees for its measure, as the Angle *A B C*.

16. An *Oblique Angle* may be *Acute* or *Obtuse*.

17. An *acute Angle* is that which is less than a *right Angle*, or 90 Degrees, as the Angle *E B C*.



18. An *obtuse Angle* is that which is greater than a *right Angle*, as the Angle *D B C*.

19. *Right lin'd Figures* are such, as are contained under *strait Lines*.

20. Of *three sided Figures*, that is, an *Equilateral Triangle*, which hath the three sides equal to each other, as the Figure *A*.



21. An *Isoceles Triangle* hath only two of the sides equal, as the figure *B*.



22. *Triangles* have also receiv'd Names from the nature of their Angles ; as if the Triangle has one *right Angle*, it is call'd a *right Angled Triangle*, as the figure *B A C*, In which *B A* is termed the *Base*, *C A* the *Perpendicular*, and the side *B C* that subtends the *right Angle* the *Hypotenuse*.

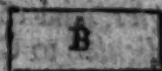


23. If a *Triangle* has no *right Angle* in it, 'tis termed an *oblique angled Triangle* ; and if such Triangle have one *obtuse Angle*, 'tis call'd an *obtuse angled Triangle* : But if no *obtuse* or *Right Angle*, then an *acute angled Triangle*.

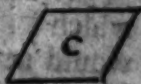
24. Of *Figures* which have four sides, that is a *Square* which hath all its sides equal, and Angles right, as the Figure A.



25. A *Rectangle*, or *long Square*, hath all four Angles right and opposite sides equal, as the Figure B.



26. A *Rhombus* hath all its sides equal, and opposite Angles equal, but none of them right, as the Figure C.

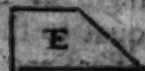


27. A *Rhomboid* hath only its opposite sides and Angles equal, but none of the Angles right, as the Figure D.



28. A *Parallelogram* is a four sided Figure, whose opposite sides are *parallel*, as the Figure A. B. C. and D. in the 24, 25, 26 and 27 *Definitions*.

29. All other *four sided Figures* are call'd *Trapezias*, as the Figure E.



30. Figures having above four sides, and those unequal, are called *many sided* (or *Irregular*) *Polygons*, as the Figure G.



31. If *Figures* have above four sides, and those equal, they are called *Regular Polygons*, as the Figure I, of five equal Sides is term'd a *Pentagon*.



32. If a *Figure* has six equal sides, 'tis named a *Hexagon* or *Polygon* of six equal sides; if seven a *Heptagon* or *Polygon*, of seven equal sides, &c.

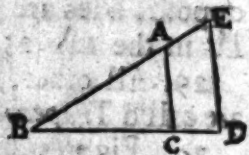
33. If a straight Line AB fall upon another straight Line CD , and make the Angles ABC , ABD equal to each other, then those equal Angles are called *Right Angles*, and the straight Line AB is termed a *Perpendicular* to CD ,



34. *Parallel* or *Equidistant* Right Lines, are such which, if infinitely extended, would never meet; such are the Lines WX , YZ .



35. *Similar* or *Equi-angled* Figures are such which have the Angles of the one severally equal to the Angles of the other, having also the sides about those equal Angles proportional; as in the two Triangles BAC , BED , the Angle B is common, the Angle at E in the Greater is equal to the Angle at A in the Lesser, and the Angle D in the Greater is equal to the Angle C in the Lesser: Also $BE : ED :: BA : AC$. So likewise, in all *Quadrangular* or *Multangular* Figures, tho' their Angles are equal, yet the Figures are not similar, unless their Corresponding Sides are proportional.



36. *Complements* of Angles or Arches are so called in reference to a Quadrant or Semi-circle, as the Angle EAB is the Complement of the Angle EAD to a Quadrant; but of the Angle EAC to a Semi-circle.



37. The *Diagonal* of any Figure is a Right Line, drawn from the opposite Angles thereof; as in the annexed Trapezium the *Diagonal Line* is BD .



38. If a Point B be taken in the Circumference of a Circle, and from it Right Lines BA , BD be drawn to the Ends of the Right Line AD , which cuts off a *Segment*, then the Angle ABD contained between the Lines BA , BD , is said to be an Angle in a *Segment*.



39. *Segments* are said to be Similliar, when the Angles so Inscrib'd are equal.

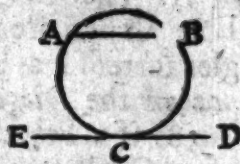
40. A *Right-lin'd Figure* is said to be *inscrib'd* in a *Right-lin'd Figure*, when all the Angles of the Figure inscrib'd touch all the Sides of the other, as the Triangle DEF is inscrib'd in the Triangle ABC . Also the said Triangle is inscrib'd in the Circle $DE F$, for the Periphery thereof touches all the Angles of the Figure.



41. A *Right-lin'd Figure* is said to be *describ'd* about a *Right-lin'd Figure*, when every Side of the Circumscribing Figure touches every Angle of the Figure about which it is circumscrib'd; as the Triangle ABC circumscribes the Triangle DEF , also the Circle DEF circumscribes the Triangle DEF .

42. A Right Line is said to be *apply'd* in a Circle, when the Ends thereof fall upon the Circumference; as the Line AB is *apply'd* in the Circle ABC . The Line AB is also called an *Inscript*; and when so *apply'd*, is said to *cut* the Circle.

43. A *Right Line* is said to *touch* a *Circle*, when touching the same, and being produced, it cutteth it not; so the *Right Line* ED , toucheth the *Circle* ABC ,



44. In a *Circle* $CGDH$, *Right Lines* CD , GH are said to be *equally distant* from the *Centre* B , when the *Perpendiculars* BA , BE , drawn from the *Centre* B , to them are equal; and that *Line* IK is said to be *farthest distant* from it, on which the longest *Perpendicular* BF falleth.



45. The *Height* of any *Figure*, is a *Line* drawn from the top of it perpendicular to the *Base*; as CG is the height or true breadth of the *Rhomboides* $BCEF$. It is also the height of the *Triangle* BCD .



What Proportion is.

46. *Proportion*, *Ratio*, or *Reason* is the *Habitude* or *Relation* that one *Magnitude* hath to another according to *Quantity*: As, if 7 be compar'd to 3, then the *Respect* that the *Quantity* of the *Magnitude* 7 has to the *Quantity* of the *Magnitude* 3, is the *Reason* or *Habitude* of 7 to 3. In every *Ratio* observe, that that *Quantity* which has *Reference*, or is *Referr'd* to another, is called the *Antecedent* of the *Ratio*; and the *Quantity* to which it is *referr'd*, is called the *Consequent*; as in the *Comparison* of 7 to 3, 7 is named the *Antecedent*, and 3 the *Consequent*.

When

When Quantities have a Reason or Proportion to one another.

47. *Quantities* are said to have a *Reason* or *Proportion* to one another, when being multiply'd, they exceed another; now there is no *Line* how short soever, but if multiply'd, may exceed a longer *Line* given. But if (any *Superficies*, as) a *Square* and *Right Line* be proposed, the *Line* cannot be so augmented in length as that it can be said to exceed the *Square*, because if infinitely increased, it hath still no breadth. Hence therefore 'tis clear, all *Homogeneous Magnitudes*, i. e. *Magnitudes* of the same kind, have a *Reason* or *Proportion* one to another.

When Quantities have the same Proportion.

48. *Numbers* or *Quantities* are said to have the same *Reason* or *Proportion*, when each *Consequent* is a like part of its respective *Antecedent*; for no *Quantity* can be said to be *big* or *little*, but as it is compar'd to another. Suppose these four *Numbers* 9. 3. 6. 2. be proposed, then if 3 is contain'd the same *Number* of times in 9, as 2 is in 6, I say they are *proportional*; that is, they are *Quantities* that have the same *Proportion*; for there is the same reason of 9 to 3, as of 6 to 2. Hence note, that the *Quantity* of any *Ratio* is known by dividing the *Antecedent* by the *Consequent*, as the *Quantity* of the *Ratio* of 7 to 3 is express'd thus $\frac{7}{3}$, or of the *Ratio* of a to b thus $\frac{a}{b}$; wherefore often, for *Brevity*'s sake, the *Quantities* of *Ratio* in the following *Theorems* are denoted after this manner, $\frac{a}{b} >$ or $\frac{a}{b} =$ or $\frac{a}{b} <$; that is, the *Ratio* or *Proportion* of a to b is greater, equal,

equal, or less than the *Ratio* of *c* to *d*. From this *Definition* 'tis evident, if *a* contains *b* as oft as it contains *c*, then *b* and *c* are equal; also if *a* contains *b* as often as *c* contains *b*, then *a* and *c* are equal.

Of the Number of Terms in a Proportion.

49. *Proportion* consisteth of three *Terms* at the least, the second whereof doth supply the place of two; and therefore that which I now call *Proportion*, is more rightly term'd *Proportionality*, or the *Similitude* or *Likeness* of *Proportions*; for as 4 to 6, so is 6 to 9; that is, the *Reason* of a first *Magnitude* compar'd to a second, must be like that of a third compar'd to a fourth: So that when there are but three *Terms*, the *Consequent* of the first *Ratio* is taken for the *Antecedent* of the second, for as $9:6::6:4$; *Proportion* it self generally denoting no more than the *Ratio* betwixt two *Quantities*.

50. One *Reason* is said to be *greater* than another, when one *Antecedent* contains its *Consequent* more times than the other *Antecedent* doth his *Consequent*: As if (in the Comparison of these, *A. B. C. D.*) *A* contains *B* more times than *C* doth *D*, then there is a *greater Reason* of *A* to *B* than of *C* to *D*.

51. An *Aliquot Part* is a lesser *Number* in respect of a greater, when it measures it exactly: As, 2 is an *Aliquot Part* of 6, because it is contain'd just three times in it; and for the same *Reason* 3 and 4 are *Aliquot Parts* of 12, for that they are precisely contain'd in it.

52. An *Aliquant Part* is a lesser *Number* in respect of a greater, when it doth not measure it exactly: As, 3 is an *Aliquant Part* of 7, because it is not contain'd precisely any *Number* of times in 7, there

there being above 2 times 3 in it, and yet not 3 times : Also, for the same Reason, 2 is an *Aliquant Part* of 9.

53. When the *Antecedent* is *Double, Treble, Quadruple, &c.* of the *Consequent*, 'tis term'd *Double, Treble, Quadruple, &c. Reason* ; and these Names do rather depend upon the *Antecedent*, than the *Ratio* it self.

54. *Numbers* are in *Continued* or *Continual Proportion*, when the intermediate *Numbers* betwixt the first *Term* and last are taken twice, that is, as *Antecedent* to *Consequent*, so is the *Consequent* taken as *Antecedent* to a fourth *Number* or *Quantity* ; so 16, 8, 4, 2 are in *Continued Proportion* for as $16 : 8 :: 8 : 4 :: 4 : 2$.

55. *Duplicate, Triplicate, &c. Ratio*, is, when the *Antecedent* is compounded of like *Reasons* : As, Let those four *Numbers* $324 : 108 : 36 : 12$ be in *continual Proportion*, then the *Reason* of the first to the third is *Duplicate* to that of the first to the second : Also the *Reason* of the first to the fourth is *Triplicate* to that of the first to the second : Dr. Barrow expresses it thus, $\frac{324}{36} = \frac{324}{108}$ twice, and $\frac{324}{12} = \frac{324}{108}$ thrice ; which denotes that the Square of the Quote of 324 by 108, is equal to the Quote of 324 by 36 ; and the Cube of the Quote of 324 by 108, is equal to 324 by 12 : So that *Duplicate Ratio* is no more than the *Proportion* of the first to the third, in three *continual Proportionals* ; and *Triplicate Ratio* the *Proportion* of the first to the fourth, in four *continual Proportionals*.

56. *Homologous Sides* are those in any *Figure* which correspond one to another, in *Numbers* or *Quantities* ; the two *Antecedents* are *Homologous Terms*, and so are the two *Consequents*.

57. *Reciprocal Figures* are those, whose Parts are so compar'd, that the *Antecedent* of one Reason and the *Consequent* of the other shall be found in the same Figure; that is, when the *Proportion* beginneth and endeth in the same Figure. In Quantities, 'tis when it beginneth and endeth in the same Rank; as if $\left\{ \begin{array}{l} a : b :: c : d \\ e : b :: c : f \end{array} \right\}$ then reciprocally, as $e : a :: d : f$.

Of the Varieties of Proportion.

58. *Alternate Reason*, or *Proportion*, is the Comparing of *Antecedent* to *Antecedent*, and *Consequent* to *Consequent*; as if $A : B :: C : D$, then alternately compar'd, 'twill be as $A : C :: B : D$.

59. *Inverse Reason* is, when the *Consequent* is taken as the *Antecedent*, and so compared to the *Antecedent*, taken as *Consequent*; as if $A : B :: C : D$, then inversly, as $B : A :: D : C$.

60. *Compound Reason* is, when the Sum of the *Antecedent* and *Consequent* is compared to the *Consequent* it self; for if $A : B :: C : D$, then by *Composition of Reason*, as $A + B : B :: C + D : D$.

61. *Divided Reason* is, when the *Excess* wherein the *Antecedent* exceeds the *Consequent*, is compared to the *Consequent*; as if $A : B :: C : D$, then by *Division of Reason* 'twill be as $A - B : B :: C - D : D$.

62. *Converse Reason*, or *Proportion*, is the Comparing the *Antecedent* to the *Excess*, wherein the *Antecedent* exceeds the *Consequent*; as if $A : B :: C : D$, then by *Conversion* as $A : A - B :: C : C - D$.

63. *Mix'd Reason* is the Comparing the Sum of the *Antecedent* and *Consequent* to the Difference of the *Antecedent* and *Consequent*; for if $A : B :: C : D$, then mixedly as $A + B : A - B :: C + D : C - D$.

64. *Proq-*

64. *Proportion of Equality* is, when there are taken more *Magnitudes* than two in one Order, and also as many *Magnitudes* in an other Order, then by comparing two to two, if there be the same Reason of *A* to *B* as of *E* to *F*, also of *B* to *C* as of *F* to *G*, and of *C* to *D* as *A. B. C. D.* of *G* to *H*, I say then by Equality of *E. F. G. H.* Proportion there will be the same reason of *A* to *D* as of *E* to *H*. Or it is a *Comparison* of the *Extream Terms* in both Ranks.

65. *Right Line* is said to be cut in *Extream* and *Mean Proportion*, when as the lesser part of the said Line is to the greater, as the greater is to the whole Line; that is, a Rectangle made of the whole Line and lesser Segment, is equal to the Square made on the greater Segment.

Of Solids.

65. A *Solid* is that which hath Length, Breadth and Thickness.

67. The *bounds* of a *Solid* are *Superficies*.

68. A *Right Line* is *Perpendicular* to a *Plain*, when it's Perpendicular to all the *Lines* it meeteth in the said *Plain*.

69. One *Plain* is *Perpendicular* to another, when a *Line* drawn in one of them is Perpendicular to the other *Plain*.

70. The *Common Section* of two *Plains* is a *Line* in both *Plains*, or it is that *Line* conceived to be drawn on one *Plain*, by the entrance of another into it.

71. The *Inclination* of one *Plain* to another, is the *Acute Angle* contain'd by two *Lines* drawn in each *Plain*, Perpendicular to the *Common Section*.

72. *Parallel*

72. *Parallel Plains* are such, that if infinitely continu'd will never meet.

73. *Like Solids* are terminated under like Plains, equal in Number.

74. *Equal Solids* are terminated under the same Number of like and Equal Plains.

75. A *Solid Angle* is made by the meeting of more than two Lines in a Point.

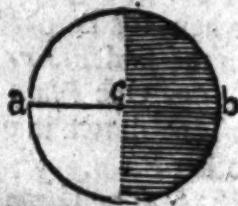
76. A *Pyramid* is a solid Figure, whose Base may be Triangle, Square, Parallelogram, or any other Polygon, but its sides plain Triangles, whose several Tops meet in a Point, as the Figure A.



77. A *Prism* is a solid Figure, containing five Plains at the least, the two opposite, (*viz.* *b* and *c.*) of which are *like, equal, parallel,* and *alike scituate*, as the Figure B.



78. A *Sphere* is a solid Body, produc'd by the moving round a *Semicircle*, upon its *Diameter* *a b*, supposed to be fix'd as the Figure C, and is therefore bounded by one Surface, to which Lines drawn from the Center or Middle of its Body, are equal.



79. The *Axis* or *Diameter* of the *Sphere*, is a straight Line passing through the Center, or that fix'd *Right-Line*, about which the *Semicircle* is moved; as *a b*.

80. A *Cone* is a round *Pyramid* in form of a *Sugar-Loaf*, generated by the turning round a *Right Angled*

Angled Triangle aec , upon one of the sides ce , that includes the Right Angle, as the Figure D.

81. The *Axis* of a *Cone*, is that fixt Right Line, ce , about which the Triangle is moved; The *Base* of the *Cone* the Circle describ'd by the moveable Right Line ea . And the *Vertex* the *Terminating Point* c .



82. A *Cylinder* is a Figure like the *Rolling-Stone* of a Garden, and is formed by the moving round a Right Angled *Parallelogram*, upon one of its sides ac , which contains the Right Angle, as the Figure E, The *Axis* of which is the said Right Line ac , and the *Bases* of the *Cylinder* the two Circles describ'd, by the motion of the two opposite sides.

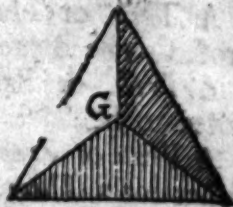


83. Like *Cones* and *Cylinders* are such whose *Axes* and *Diameters* of their *Bases* are proportional.

84. A *Cube* or *Die*, is a solid Figure contained under 6 equal Squares, as the Figure F.



85. A *Tetraedon*, is a solid Figure contain'd under four Equilateral Triangles, as the Figure G.



86. An *Octaedron*, is a solid Figure contained under eight Equal and Equilateral Triangles, as the Figure H.



87. A *Dodecaedron*, is a solid Figure contained under twelve E-

qual

qual Equilateral and Equiangular Pentagons, as the Figure I.

88. An *Icoſedron*, is a ſolid Figure contained under twenty Equal Equilateral Triangles, as the Figure K.



89. A *Parallelepipedon*, is a ſolid Figure contained under fix *Quadrilateral* Figures, whereof thoſe which are oppoſite are parallel, as the Figure L. This Figure does alſo come under the 77th Definition.



90. A *Solid Figure*, is ſaid to be *Inſcribed* in a ſolid Figure, when all the Angles of the Figure Inſcribed are comprehended either within the Angles, or in the Plains of the Figure wherein it is Inſcrib'd.



91. A *Solid Figure* is ſaid to be *Circumſcrib'd* about a *Solid Figure*, when either the Angles or Sides, or Plains of the Circumſcrib'd Figure touch all the Angles of the Figure it contains.

Terms Explain'd.

A *Theo.* or *Theorem* is when ſomething is propoſed to be demonſtrated.

A *Prob.* or *Problem* is when ſomething is propoſed to be done.

A *Prop.* or *Propoſition* is uſed promiſcuouſly, *i. e.* either for a *Theorem* or a *Problem*.

A *Def.* or *Definition* is the unfolding or *Explicating* of the *Nature* and *Affection* of a thing.

A *Poſt.* or *Poſtulate* is a grantable requeſt, or ſuch a *Demand* as reaſonably cannot be denied.

An

An *Ax.* or *Axiom* is a *Principle* in any Art, so evident, that it needs nothing but the light of Reason to demonstrate it.

A *Cor.* or *Corollary* is a consequent truth, gained from some preceeding Demonstration.

A *Schol.* or *Scholion* is a short Critical Exposition, gained from a former Demonstration, or a *Corollary* wanting an *Explication*.

Hip. or *Hypothesis* is when a thing is supposed, or given so to be, as if it be said A is equal to B by *Hypothesis*; it is as much as to say A is equal to B by Supposition.

Con. or *Construction* is the drawing of Lines and framing of Figures, or a preparing the Proposition for a Demonstration.

Dem. or *Demonstration* is the proving a thing by *Definitions* and *Axioms*, and so from several Arguments drawing a Conclusion, that it has that Affection the Proposition did assert.

Lem. or *Lemma* is the Demonstration of some Premise, in order to shorten a following Demonstration.

Postulates, or grantable Demands.

1. It may not be denied, but that a line from any point to any point may be drawn.
2. That a line may be produced to what length you please.
3. That from a given Center, a Circle may be describ'd with any distance.

Axioms.

1. Things equal to the same third, or equal things, are also equal one to another.

2. If

2. If two equal things, you add the same, or equal things, the wholes shall be equal.

3. If from equal things, the same or equal things be taken away, the Remainders shall be equal.

4. Things Double, Treble, Quadruple, &c. or a Half, a Third, a Quarter, &c. of the same or equal Things, are also equal one to another.

5. The whole is greater than any of its parts, but equal to all of them taken together.

6. Those things which fit one another, and agree in all their parts, are equal.

7. If one whole be double, treble, &c. to another, and that which is added to, or taken away from the first double, treble, &c. to that which is added to, or taken away from the Second, the Sum or Remainder of the First, shall be double, treble, &c. to the Sum or Remainder of the Second.

8. Two right lines cannot have one and the same Segment common to them both.

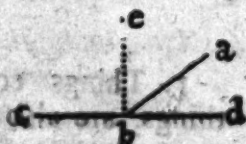
9. Two right Lines include no space.

Theorems in Plain Geometry.

THEOREM I.

IF a strait Line ab fall on another strait Line cd , it makes either two right Angles, or two Angles equal to two right.

Dem. If ab is perpendicular to cd , the $\angle abd = abc$ (def.



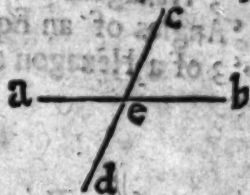
33.) and therefore both right, If not, let eb be a perpendicular, then is the Angle $abd + abe$ equal to one right (def. 33. ax. 5.) to which add the Angle ebc , and they all make two right, Which was to be demonstrated.

Cor. 1. Hence, if never so many right lines, as ca, da, ea, fa, ga , meet together at one point, and on the same side of any straight line as bb , all the Angles form'd by them at that point, are only equal to two right (ax. 5.)



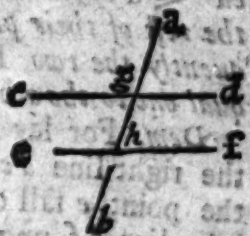
2. If to any straight line, as ea and point therein a , two right lines ba, ab be drawn on contrary sides, making the Angles bae, eab equal to two right, then shall those lines make one straight line.

Schol. 1. From this Theo. 'tis evident, that if two right lines cut through each other, the opposite Angles at the point of cutting are equal: For the $\angle aec + aed$ is equal to two right; by the preceding Theo. so also is the $\angle aed + deb$; therefore (ax. 3.) $\angle aec = deb$ W. W. D.



Cor. Hence all the Angles made about one point, are equal to four right.

Schol. 2. If a Right line cut through two parallel right lines, 'tis manifest 'twill make the following Angles equal; that is the Angle $f h g = c g h$, and the Angle $ag d = a h f$. For two parallel lines are but the bounds or outides of one broad line, and therefore



from

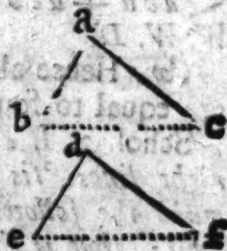
from the foregoing Demonstrations, 'tis evident that the $\angle fbg =$ to $cg b$, and the $\angle ag d = abf$ W. W. D.

Use. This Theo. is of great use, both in Plain and Spherical Trigonometry, for when one of the Angles made by the falling of a perpendicular is known, the other is also known: For Example, if the Angle $ag d$ in the last Figure be 60 degrees, the $ag c$ is 120 degrees, the Complement thereof to two right, or 180 degrees, which is (as hath been proved) the quantity of both these Angles, and is therefore found by subtracting 60 degrees therefrom.

Also from the last Cor. is determined what Polygons may be joined together for Paving, and by it we find that either 6 Triangles, 4 Squares or 3 Hexagons will perform the business; for the 6 Angles of an Equilateral, the 4 of a Square, and 3 of a Hexagon are equal to 4 right.

THEO. II.

If in two Triangles abc , def there be two sides ab , ac of the one, equal to two sides de , df in the other respectively, and the included $\angle a$ of the one, equal to the included $\angle d$ of the other, then shall the rest of their parts, and consequently the two Triangles be equal one to the other.



Dem. For lay the point d on the point a , and the right line de on the right line ab , then shall the point e fall on b , for $de = ab$ (Hyp.) and the right line df upon the right line ac , because the

Angle

Angle $b a c = e d f$ (*Hyp.*) also the point f will fall on the point c , for $d f = a c$ (*Hyp.*) and therefore (*ax. 6.*) will the rest of the parts be equal, that is $\angle a b c = d e f$ and $\angle a c b = e f d$, also $b e = e f$, and consequently both Triangles, *W.W.D.*

Schol. After the same manner it might be prov'd, that if either two Angles and a side comprehended, or all three sides, be equal to the three respective parts or sides of another Triangle, then shall the two Triangles be equal to each other.

Use, Hence may an inaccessible distance (suppose $a d$) be measur'd, for from a erect $a c$ as long as you please, perpendicular to $a d$, join $c d$, on the point c describe the prickt arch to measure the $\angle$



$a c d$, and make $a c b$ equal thereto, then draw $c b$, which meets the line $d a$ produc'd in b , now because in the two Triangles $b a c$, $d a c$, there is a common side $a c$, and two Angles including this side in one, viz. $b a c$, $b c a$ equal to the two Angles $c a d$, $a c d$ including the said side in the other, therefore shall the rest of the parts be equal, and consequently $b a = a d$, so that measuring the accessible line $b a$, you have the length of the inaccessible line $a d$.

Also by this *Theo.* we are taught how to strike a Bowl at *Billiards*, so that by its reflection, it will hit any other Bowl propos'd: Suppose one Bowl at the point a , and the other you would hit at b , and $c d$ the *Billiard Table* imagine to the point b , the Line $b e$ be drawn perpendicular to $c d$, will $d e = d b$, and join $a e$, then I say that f is



C

the

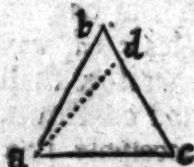
the point, to which if the Bowl at a be directed, it will by reflection carry it to b . For in the Triangle $b f d$, $e f d$ there are two sides and an Angle respectively equal, but (by *Schol. 1. Theo. 1.*) the $\angle a f c = e f d$, but the $\angle e f d = d f b$, and therefore the $\angle d f b = a f c$, and therefore the point of the Table is f , which if struck from a will reflect the Ball to b .

Likewise from hence it is easie to conceive, how Light may be transferr'd from one place to another, *viz.* by only placing several Glasses, so that the Angle of Incidence may be equal to the Angle of Reflection, by which means the Light of a Candle may be brought out of one Room into another.

THEO. III.

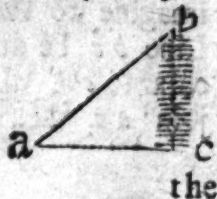
In any plain Triangle as $a b c$, equal Angles as a and c are subtended by equal sides.

Dem. If you deny it, let one side be longer than the other, suppose $c b > a b$, make $c d = a b$ by drawing the line $a d$. Now because in the Triangles $b a c$, $d a c$ there is a common side $a c$, also $c d = b a$ and the contained Angle $d c a = b a c$, therefore (*Theo. 2.*) is the Triangle $a d c = a b c$ a part to the whole, which is impossible.



Cor. Hence if in any Triangle the three Angles are equal, then are the three sides also equal, and *contra.*

Ufe. By this *Theo.* the height of any Object from its shadow may be found; for watch till the Sun hath 45 Degrees of height, and at that Instant measure the length of the shadow of any Object, and

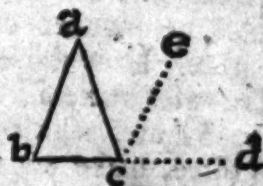


'the length will be equal to the height of the Object, for the Angles a and b made by the Sun's Ray ab , with the Object bc and ground line ac are equal, and therefore (by this *Theo.*) the sides ac , ab are equal that subtend them.

THEO. IV.

The three Angles a , b , c of any Plain Triangle, as abc are equal to two Right.

Dem. Produce any side as bc , and draw ce parallel to ab , then is the Angle $ace = bac$ (*Schol.* 2. *Theo.* 1.) and by the same the $\angle ecd = abc$, now if to the whole Angle acd you add the Angle acb it is evident (*Theo.* 1. *ax.* 1.) that the three Angles, viz. $a + b + acb$ are equal to two Right. *W.W.D.*



Cor. 1. Hence it follows, that if one side of a Triangle is produced, the outward Angle formed thereby, is greater than either of the inward opposite Angles of the Triangle.

2. That if one Angle in a Triangle is right, the other two taken together are equal to a Right, but severally acute.

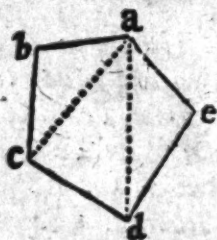
3. That if one Triangle hath two Angles severally or together, equal to two Angles in another Triangle (severally or together) then shall the remaining Angle of the one, be equal to the remaining Angle of the other; so also if two Triangles have one Angle of the one, equal to one Angle of the other, then is the Sum of the remaining Angles in the one, equal to the Sum of the remaining Angles in the other.

4. All the Angles of an Equilateral Triangle, and the two Angles at the Base of an Isosceles Triangle are acute.

5. That one Angle of an Equilateral Triangle is $\frac{1}{3}$ of one right Angle, for $\frac{2}{3}$ of two right Angles, is $\frac{2}{3}$ of one right.

Schol. 1. *By the help of this Theo. you may know how many Degrees there are in any right lin'd Figure.*

Suppose in the Figure *abcde*, first by Diagonals cut the Figure into Triangles, which is three, for the number of Triangles is always lesser by two, than the number of sides in the Figure. Now the Angles of the Triangles make all the Angles of the Figure, which because there is three, make Six right Angles by the preceding Theo. which 6 being multiplyed by 90, the degrees in one right Angle will give you 540, the Degrees in the whole Figure.



Cor. Hence all right lin'd Figures of the same number of sides, have the Sums of their Angles equal.

Schol. 2. *Hence may the Degrees of the Angle of any Regular Polygon be known:* For dividing the number of Degrees in all the Angles of a Polygon, by the number of the Angles, the Quote will give you the Degrees contained in one Angle; thus proceeding, you will find the Angle of an Equilateral Triangle 60 Degrees, the Angle of a Square 90, the Angle of a Pentagon 108 Degrees.

Use. Many and great are the uses of this Theo. for 'tis by this Astronomers determine the Parallax, which is an Angle subtended by the Semidiameter of the Earth. As let *a* denote the Earths Center, and from *b*, a point in its Surface, let be taken by Observation the Angle *dbc*, which is the distance of the Star from the Zenith. Now 'tis plain, that if the Earth were transparent, and the
Star

Star observed from a its Center, the Angle $c a d$ would be made, which is less than the Angle $c b d$, by the Angle $a c b$, for $c b d = c a d + a c b$, by the preceding Theo. so that the Angle $a c b$ will be equal to the excess of the Angle $c b d$ above the Angle $c a d$. If therefore I can by any Artifice find how far distant from the Zenith the Star ought to appear at the Center of the Earth, at the same time 'tis observ'd from the Surface, I may then find the Quantity of the Angle $a c b$ which is the Angle of Parallax, and is subtended by $b a$ the Semidiameter of the Earth.



Again, by the Second Schol. Surveyors have a method to prove whether (when in Surveying a Field by going round it) they have taken the Angles right, for having observed them by an Instrument, find their Sum by adding them together; for Example, let the Figure $a b c d e$ (Schol. 1.) be a Field, whose Angles had been taken: divide the Figure into Triangles, and find by the said Schol. the number of Right Angles contained in the Figure, then if the Sum of those so found, be equal to the Sum of those which were taken, the Work is right, otherwise not.

Lastly, By it may be Constructed any Regular Polygon upon a given Line, which is of great use in Fortification; for in the Ichnographick Projection of any Regular Fort, the first thing to be done is to describe a Regular Polygon upon a given side.

THEO. V.

The greatest side $c b$ of any Triangle $c a b$, subtends the greatest Angle a .

Dem. Take from cb , $cd = ca$ and join ad . The Angle $cda = cad$ (*Theo. 3.*) But (*Theo. 4. Cor. 1.*) the Angle $cda < cba$, that is the Angle $cad < cba$; and therefore the Angle $cab < cba$ (*ax. 5.*) In like manner it may be proved that the Angle cab is greater than the Angle c . *W.W.D.*



Cor. Hence the greatest Angle will always be subtended by the greatest side.

Use. The principal use of this *Theo.* is to prove that from one and the same point, and to the same right line, there can be but one perpendicular drawn, and also that this perpendicular is the shortest line or nearest distance from that point to the right line to which it is a perpendicular; For Example, let ca be perpendicular to bd , I say then, that ca will be less than cb , because it subtends a lesser Angle; For the Angle bac being right, the Angle b is acute, (*Theo. 4. Cor 2.*) and therefore $ca < cb$. After the same manner it might be proved, that ca will be less than any other line drawn from c to any point in the line bd , except a .



Also from hence it may be proved that a Bowl exactly round cannot rest on any plain parallel to the Horizon, but upon one certain point; for let abc represent the Surface of the Earth, and d its Center, then let any plain suppose fg be a Tangent to the Earths Surface. Now because 'tis the nature of all heavy Bodies not to rest when they may descend, and the Bowl at g having nothing

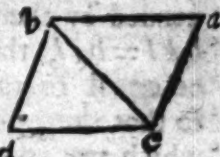


' thing to hinder its nigher approach to the Center,
 ' must (by consequence) continually descend till it
 ' comes to the point *b*, where it will rest, because
 ' it is then nigher the Center *d*. Hence it is also
 ' evident, that a liquid Body must flow from *g* to *b*;
 ' and that the surface of all Stagnant Liquors is
 ' Spherical.

THEO. VI.

*The opposite sides ab , dc and bd , ac in a pgrm.
 are equal, and the opposite Angles a , d , and abd ,
 acd , are also equal, and the Diameter bc bisects it.*

Dem. Because (def. 28.) ab , cd are parallel, there-
 fore (Theo. 1. Schol. 2.) the $\angle$
 $abc = dc b$, also for the same
 reason the Angle $bca = dbc$,
 and the side bc common, there-
 fore (Schol. Theo. 2.) the Tri-
 angle abc will be equal to bcd , and consequently
 the side $ab = dc$ and $ac = bd$. *W.W.D.*

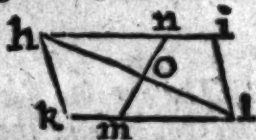


Schol. Hence it might easily be proved that if two
 equal and parallel lines, (suppose ba , dc in the last
 Figure) be joined together with two other right lines
 (ac , bd) then are those Lines also equal and parallel.

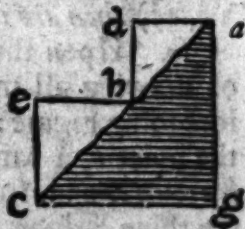
Use. Upon this Theo. is grounded the method
 of drawing all manner of parallel lines as in the
 foregoing Figure: Let it be required to draw through
 the point *b*, a parallel to dc ; join bd , from *b* draw
 $ba = dc$, and from *a* let be drawn $ac = bd$ and
 the thing is done, for because bd , and ac are equal,
 and $ba = dc$, therefore (preceding Theo.) $bcd a$ is
 a pgrm. and therefore hath its opposite sides ba ,
 dc parallel.

Also from hence a Method is learnt of dividing
 any pgrm, into two equal parts, by a line that shall

pass through any point within the Figure (which Surveyors have sometimes occasion to perform, in dividing a peice of Land into two equal parts, yet so as the Fence may pass through a Pond or Lake which is in some part thereof.) As let it be required to divide the *pgrm. b i k l* into two equal parts by a line that shall pass through the point *m*, divide the Diameter equally in *o*, and draw the right line *mon*, this Line shall divide the said Figure into two equal parts. For the Triangle $bno = lmo$, there being in either, two Angles and a Side comprehended by them severally equal, also the Triangle $bkl = hil$, therefore shall the Trapezium $bmnk$ be equal to the Trapezium $nilm$ (ax. 2 and 3.) so that the *pgrm. b i k l* is bisected by the line *mn*, which was to be done.



Lastly. By the *Schol.* of this *Theo.* may be measured the Horizontal Lines and Perpendicular Heights of *Mountains*, which Lines are hidden by their *solidity*. For Example, let the Mountain *agc* be to be measured; take a very large square as *abd*, and placing one end thereof at *a* in such fort, that the other side *db* may be perpendicular to the Horizon, then measure the sides *ad*, *db*; do the like again at the point *b*, and measure the lines *be*, *ec*, then if you add the sides (*da*, *eb*) which are parallel to the Horizon together, the Sum will give the length of the Horizontal Line *cg* and the sides (*bd*, *ec*) perpendicular to the Horizon being added, gives the perpendicular height *ag*. Note, if the

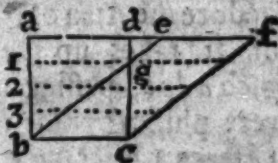


'side of the square da be made to slide higher or lower on the side db , the thing is practicable.

THEO. VII.

Parallelograms bd , bf standing on the same (or an equal) base bc , and betwixt the same or equidistant Parallels af , bc , are equal.

Dem. For $ab = dc$ (Theo. 6.) and $ad = bc = ef$ by the same, add de common, then is $ae = df$ (ax. 2.) Also the Angle $eab = fdc$, therefore the Triangle $fae = eab$, take away the common Triangle dge , there remains the Trapezium $cgfe = adbg$ (ax. 3.) add the common Triangle bgc , then is the pgrm. $bd = bf$ (ax. 2) W.W.D.



Cavalierius his Demonstration of the former Proposition by the Method of Indivisibles.

LET the former pgrms. bd , bf be proposed having the same Base bc , and being betwixt the same parallels af , bc ; divide the side ab in as many parts as you please, suppose into 4, and from those points draw lines parallel to the Base, which produce to the out side cf of the pgrm. bf . Now 'tis plain that there will be no more lines in the one than there is in the other, neither will they differ in length, being all equal to the Base bc , nor will they be closer in the one than in the other, for the lines are all parallel to the Base, and consequently one to another, therefore will the two pgrms. be equal, because compounded and made up of like and equal parts.

Use.

Use. This *Theo.* presents us with the method of measuring any *pgrm.* (whether Square, Rectangle, Rhombus, or Rhomboids, for they all come under that Denomination (by their *Def.* and *Theo.* 6.) as thus. Imagine in a Rectangled *pgrm.* the side *ab* to be carried perpendicularly through the whole Line *bc*, or else *bc* through *ab*, then shall this motion produce the Area of the *pgrm.* for it takes up the whole space *adcb*, passing through every phisical point therein.



Hence a Rectangle is said to be produced by the multiplication of two Lines, as if *ab* be 5 and *bc* 3, draw 3 into 5; and 'twill produce 15 for the said rectangle.

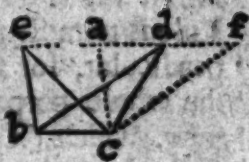
This being supposed, the Dimension of any oblique *pgrm.* as *ef* may be found, by multiplying the Base *bc* into the true height (which is a Line let fall perpendicular to the Base from some point *e*, in the opposite side) as *en*, the product of which giveth the Area of the *pgrm.* *ef*; for by the preceding *Theo.* 'tis equal to a right *pgrm.* having an equal base and height.

Thus if *en* were 6 and *bc* 5 the Area of the Rhomboids or *pgrm.* will be 30. After the same manner is the Rhombus measured, by dropping a perpendicular from an Angle to the side opposite, and multiplying any one side by the said perpendicular. The Square is measured by multiplying one of its sides into it self, as if the side of a Square be 4, the Area of it will be 16.

THEO.

THEO. VIII.

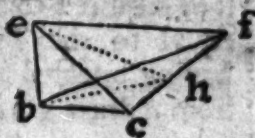
Triangles bce , bcd standing on the same (or an equal) base bc , and betwixt the same or equidistant parallels ef , bc are equal, and contrary,



i. e. equal Triangles bce , bcd on the same base (and side) bc , are also between the same parallels ef , bc .

Dem. Compleat the *pgrms.* ba , bf by drawing ca , cf parallel to be , bd . Then (Theo. 6.) ec bisects the *pgrm.* ba , as also dc the *pgrm.* bf , therefore the Triangle $bce = \frac{1}{2}$ *pgrm.* $ba = \frac{1}{2}$ *pgrm.* bf (Theo. 7. ax. 4.) $= \triangle bcd$ W.W.D.

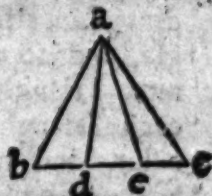
Secondly, If ef is not parallel to bc , draw eh parallel to it, and join hb , then is the $\triangle bhc = \triangle bec$ (by this Theo.) $= bfe$ (Hyp.) Which (ax. 5.) is impossible.



The like consequence would follow, if eh was drawn above ef .

Cor. Hence 'tis evident, that a *pgrm.* betwixt the same parallels, and on the same (or an equal) base with a Triangle, is double to it.

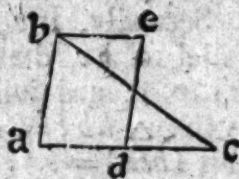
Use. * By this Theo. a practical way is found to divide a triangular Field into any number of equal parts; For Example, divide the Triangle abc into three equal parts, divide any side into three equal parts, as abc in the points d , e , and from the opposite $\angle a$ draw ad , ae , then are the Triangles bad , dae , eac equal (preced. Theo.) for they have the same height, and stand on equal Bases.



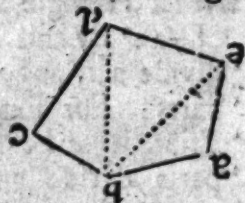
2. This

2. ' This *Theo.* doth also present us with the reason of the Rule for measuring a Triangle, which Rule is to multiply half the base by the whole perpendicular, for since (*Theo.* 7.) the whole base by the whole perpendicular produces the Area of a *pgrm.* which (*Cor.* of this *Theo.*) is double to a Triangle of the same (or an equal) base and height, therefore the whole of one Line by the half of the other, will produce but half the Area of the *pgrm.* which must therefore by the said *Cor.* be the Area of the Triangle.

3. ' From this Proposition may easily be seen how a *pgrm.* may be made equal to any Triangle, as let it be required to make a *pgrm.* equal to the Triangle abc , having bisected the base in d , from d draw a parallel to ab , making $de=ab$ and join be , then is the *pgrm.* ae equal to the Triangle abc , as is manifest from the preceding *Theo.*



4. ' It is also by this Proposition, that the measurement both of the Circle, and also of all right lin'd Figures is performed. For Example, Let the Figure $abcde$ be proposed to be measured, draw Lines from Angle to Angle, and so reduce the Figure into Triangles, then measure each Triangle severally (by letting fall a perpendicular, and measuring both it and the base by a Scale of equal parts) and collect all those Area's into one Sum, so shall this total be the Area or Content of the whole Figure.



5. ' If it were required to measure a Circle, Imagine

' gine the Circumference divided into 100000 (or
 ' more) equal parts, now I say the difference be-
 ' twixt one of those Arches and its subtense, will be
 ' so small, that scarce any operation will require
 ' greater exactness, but that the one may be taken as
 ' equal to the other, the difference being insensible :
 ' If therefore Lines are conceived to be drawn from
 ' the Center, there will be 100000 Triangles con-
 ' stituted, the Sum of the Area's of which Tri-
 ' angles will be equal to the Area of the Cir-
 ' cle, and the Sum of all the Bases equal to its
 ' Circumference, and therefore by the preceding
 ' Theo. if the half Sum of all the Bases (*i. e.* half
 ' the Circumference) be multiplied by the whole
 ' perpendicular, which without sensible Error may
 ' be taken for the Semi-diameter of the Circle, the
 ' Product will be the Area of all the Triangles, and
 ' consequently the Area of the whole Circle.

' Hence 'tis evident how a Triangle
 ' may be made equal to a given Cir-
 ' cle. For having drawn the Semi-di-
 ' ameter ba , erect from a the perpen-
 ' dicular ac , and make it equal to the
 ' Circumference of the Circle, then
 ' join the points c, b , by the right line
 ' cb , so shall this Triangle abc be e-
 ' qual to the Circle add , as may
 ' plainly appear from what before has
 ' been delivered.

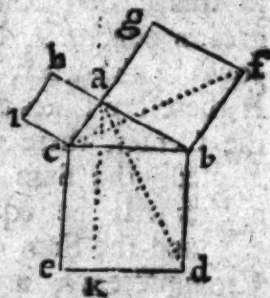


THEO. IX.

*In all Right Angled Triangles as bac , the Square
 of the Hypotenuse bc is equal to both the Squares
 hc , gb of the other two sides added together.*

Dem.

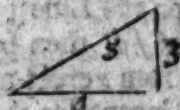
Dem. Let the Figures bc , gb , hc , be the Squares of the three sides bc , ba , ac , and let ak be drawn parallel to ce , as also the lines fc , ad . Then because the Angles cbd , $fb a$ are equal (being both right) add to each the Interjacent Angle abc , then is the $\angle fbc = abd$ (*ax. 2.*) but $fb = ab$ (*def. 24.*) and $bc = bd$, therefore the Triangles fbc , abd are equal (*Theo. 2.*) but the *pgrm.* $b k$ is double to the Triangle bad (*Cor. Theo. 8.*) also the Square bg , is double to the Triangle fbc , for gc is a straight Line, therefore is the *pgrm.* $b k$ equal to the Square bg (*ax. 4.*) After the same manner it might be proved that the *pgrm.* kc is equal to the Square ch , therefore (*ax. 2.*) the whole square bc of the side bc subtending the right Angle, is equal to both the Squares, *viz.* bg , ch made on the sides ba , ac containing the right Angle *W.W.D.*



Use. 'This *Theo.* is of such great use, not only 'in several parts of the *Mathematicks*, but also in 'many Mechanick Operations, that 'tis said *Pythagoras*, the first finder thereof, in gratitude to the '*Muses* sacrific'd a Hundred Oxen to them, he supposed it (as by this it seems) beyond the power of 'bare humane Invention; neither was this his esteem 'thereof so Irrational, as to some perhaps it may 'appear, since 'tis the foundation of many considerable *Practises* in the *Mathematicks*.

'As First, *Trigonometry* which cannot be performed without the aid and assistance hereof, for if in 'any right angled Triangle, the Hypotenuse and a 'Leg be given the other by this *Theo.* may be found, for

' for having squared the *Hyp.* which for Example, I
 ' shall suppose 5, and likewise the given Leg, which
 ' let be 3, then taking 9 the Square thereof from
 ' that of 5 which is 25, the Re-
 ' mainder will be 16, the square
 ' Root of which is 4, the length
 ' of the other side; Thus likewise
 ' if the two legs were given, the length of the *Hypo-*
 ' *tenuſe* might be found, by squaring each Leg, and
 ' adding the two Products together, to wit, 9 and 16
 ' the Sum of which is 25, and extracting the square
 ' Root of this Sum, it will give 5 the length of the
 ' Hypotenuse required.



' Secondly, 'Tis by help of this *Theo.* that the Ta-
 ' bles of natural *Signs* and *Tangents* are Calculated,
 ' by which, both plain and spherical Triangles have
 ' their Solution.

' Thirdly, By this *Theo.* we not only can make a
 ' Square, double, treble, quadruple, &c. to any one
 ' given, but also construct a square equal to any
 ' number of squares. As First, to double the square
 ' *bd* do thus, produce one side *ba* making *ea* equal
 ' to one of the sides, then join the points *de* and the
 ' thing is done, for the Square erected on the Line
 ' *de*, is equal to the square
 ' *bd*, and also the square
 ' of the Line *ac* (by the
 ' preceding *Theo.*) and
 ' therefore double to either



' of them. But if it were required to make a square
 ' equal to the Sum of two squares made on the Line
 ' *dc* and *ae*, then from *e* erect the perpendicular *ef*,
 ' and draw the right Line *df*, so shall the square
 ' made on the line *df* be equal to the Sum of the
 ' Squares made on the lines *dc* and *ef* by the pre-
 ' ceding

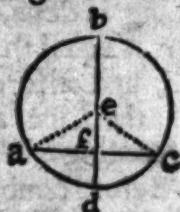
ceding *Theo.* Hence also may the subtraction of Squares be performed, *i. e.* by it may be found the difference of any two Squares, as if the Square made on the Line fe , were to be taken from the Square of fd , then by drawing a Line equal to fe , and from e erect a perpendicular de , take between your Compasses the length of the Line fd , set one foot in f , and extend the other till it fall on the perpendicular at d , then join fd and 'tis done, for since the square of the Line fd is equal to the Squares of the Line fe , ed , therefore shall the square of the Line de be equal to the difference of the squares of the Lines fd , fe .

Many other useful *Practises Mechanicks* perform by this *Theo.* as the finding the length of *Strings*, *Hip-Rafters*, *Scaling-Ladders*, and the like.

THEO. X.

If in a Circle ea cb , a right line bd drawn through the Center bisect any other right line ac not drawn through the Center, it shall cut it at Right Angles, and if it cut it at Right Angles it shall bisect the same.

Dem. Draw from the Center e the right lines ea , ec . Then because $af = fc$ (*Hyp.*) and $ea = ec$ (*def. 6.*) also the side ef common, therefore shall the $\angle efa = \angle efc$ (*Theo. 2.*) and consequently right (*def. 33.*) *W. W. D.*



Again, because the Angle $eaf = \angle ecf$ (*Theo. 3.*) and the side ef common, therefore (*Theo. 2.*) shall $af = fc$, that is ac is bisected. *W. W. D.*

Cor. 1. Hence therefore the Center of a Circle is in that Line, which bisects another at right Angles.

2. Hence

2. Hence in an *Equilateral* or *Isoceles* Triangle, if a line drawn from the Vertical Angle bisect the base, that line is perpendicular to it and contrary.

Use. By help of this *Theo.* the Center of a Circle may be found, for having applied the right line ac (See the preceding Figure) in the Circle bac , and bisect it with the perpendicular bd , which by the foregoing *Theo.* will pass through the Center, and is (def. 8.) the Diameter; If therefore the said Diameter be divided into two equal parts in e , that point of Section shall be the Center.

It also furnishes Artificers with a Geometrical way of finding a Center, whereby to strike an Arch when they have the height ab and breadth cd given, for join the points bc , bd and produce ba , then bisection cb (or db) with the perpendicular ne , producing it till it cut ba prolonged in e , which point is the Center that will sweep an Arch to the given Height and Breadth.

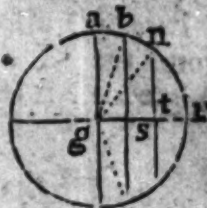


Lastly, 'Tis used in Trigonometry, for by it is proved that the sine of an Arch is perpendicular to the Diameter; It also helps to demonstrate that the sides of a Triangle have the same proportion as the sines of the opposite Angle.

THEO. XI.

In a Circle as $gac b$ the greatest Line ad is the Diameter, and the next greatest is that which is nearest the Center.

Dem. Draw the Lines gb , gc and gn . The Diameter $ad = gb + gc$ (def. 6.) but $gb + gc < bc$ (def. 2.) therefore $ad < bc$ W.W.D.



D

d c Again

Again, $bsq + gsq = gbq$ (*Theo. 9.*) $= gnq = gtq + ntq$. Take away the square of the line gs , which is but part of gt from the square of gb , there will remain ntq less than sbq , and therefore the square nt is less than sb , and consequently no less than bc . *W.W.D.*

Use. 'This *Theo.* shews us that the Chord of an Arch greater or lesser than a Semicircle, is less than the Diameter.

'*Theodesius* also makes use of it to demonstrate that in a Sphere, the lesser Circles are more remote from the Center than those that are greater.

'It is also by this *Theo.* that *Aristotle* proves that the *Rowers* in the middle of a Galley have greater strength than those that are either at the fore or hinder part thereof, for the sides of the Galley being *Belging*, the *Oars* of the middle part are longer, *i. e.* the *fulcriment* or point of bearing comes nearer the middle of the Oar, and supposing they pull with equal strength, he that sits in the middle (by this property) has his strength increased; as the Doctrine of the *Mecanicks* in the case of the *Balance* and *Leaver* doth plainly demonstrate.

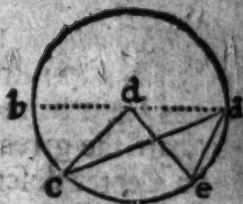
THEO. XII.

The Angle bac at the Circumference of a Circle, is equal to half the Angle bdc at the Center, standing on the same (or an equal) Arch bc of the Circumference.

Dem. First, The External $\angle bdc = dac + dca$ (*Theo. 4.*) also the Line $dc = da$ (*def. 6.*) therefore the $\angle dac = dca$, and consequently the Angle bac is equal to half the Angle bdc . *W.W.D.*

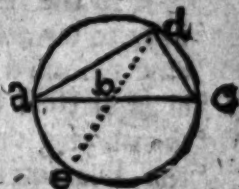
Secondly,

Secondly, The $\angle bdc$ is double to the $\angle bac$ by the foregoing case, and so also is the $\angle bde$, double to the $\angle bae$, therefore (ax. 8.) the $\angle cae = \frac{1}{2} cde$ W.W.D.



Cor. Hence it follows that the Angles subtended by (equal or) the same Arch of the Periphery are also equal, for that they are both double to the same thing, viz. the Angle at the Center.

2. That the Angle in a Semicircle is right, which (though evident from the former Theo.) may be thus demonstrated. From d through the Center draw the line de , then is the $\angle ade = \frac{1}{2} abe$ and the $\angle edc = \frac{1}{2} ebc$ but half the two Angles abe and ebc is a right Angle (Theo. 1.) therefore the Angle adc equal to one right angle. W.W.D.



Schol. By this Theo. it may easily be Demonstrated, that Angles made at the Circumference contain half the quantity of the Arch they stand on, for the Arch aeb is the measure of the Angle acb (def. 13.) but the Angle $adb = \frac{1}{2} acb$, therefore the Angle adb equal to half the Arch aeb , W.W.D.



Use. This Theo. is used in demonstrating some Trigonometrical Propositions, as also in Astronomy for finding the Apogee and Excentricity of the Sun.

Also in Opticks it shews that the Line ab will appear of the same length when seen either from d or c , because in both cases you behold it under equal Angles.



Thirdly, A very easie and practicable way is found to make an Angle equal to half of any given Angle; as let it be required to make an Angle equal to half the given Angle $b a d$, on a as a Center describe with any Radius the Circle $b c d$, and from any point (as c) in that part of the Circumference, not included by the given Angle draw the lines $c b, c d$, then is the Angle $b a d$ double to $b c d$ (by the preceding Theo.) i. e. the Angle $b c d = \frac{1}{2} b a d$.



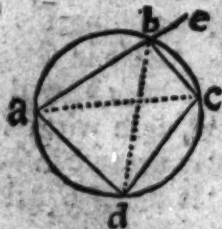
Also the second Cor. hereof shews *Mechanicks* how to try whether their Squares be true, for having describ'd a Semicircle $a d b$, they apply the head of their Square $a d b$ to the Circumference, then laying one side as $d a$ upon the extremity of the Diameter a , the other part or side, $d b$ will fall exactly on the other extreem part of the Diameter b , if the Square be true. See the Figure of the said Cor.

Again, by the aforefaid Cor. is shewn the manner of raising a Perpendicular on the end of a given Line, as also of letting fall from a Point assign'd over the end of a given line.

THEO. XIII.

The opposite Angles $a + c$ or $b + d$ of every four sided Figure inscrib'd in a Circle, are equal to two right Angles.

Dem. Draw the Lines $b d, a c$, then all the Angles in the Triangle $a b c$ are equal to two right, but the $\angle a d b = b c a$ (Cor. 1. Theo 12.) also for the same reason is the Angle $b d c = b a c$,



therefore

therefore the whole Angle $\angle a d c = b a c + b c a$ equal to the Complement of the Angle $a b c$ to two right, therefore the $\angle a b c + a d c$ is equal to two right Angles. *W.W.D.*

Cor. Hence if one side of a Quadrilateral Figure inscrib'd in a Circle be produced, the outward Angle $e b c$ is equal to the internal Angle, which is opposite to the Angle adjacent.

Use. 'This proposition is helpful in the demonstrating that the sines of the sides are in the same reason, as are the sines of the opposite Angles; 'Ptolomy also makes use of it to Calculate the Table of Chords.

T H E O. XIV.

If a right line $a b$ touch a Circle, and from the point of Contact, through the Center of the Circle, a Diameter be drawn, that Line shall be perpendicular to the Tangent $a b$.

Dem. If you deny it, let some other line $f g$ be drawn from the Center perpendicular to the Tangent, and cutting the Circle in d , therefore, whereas the Angle $f g e$ is right, (*Con.*) then is the $\angle f e g$ acute, and therefore $f e$ (that is $f d$) is greater $f g$, which is absurd.



Schol. The contrary of this is also Evident, that is, if a right line $a b$ touch a Circle, and from the point of contact, another right line $e f$ be erected at right angles, it shall pass thro' the Center, that is, it shall be the Diameter of the Circle.

Use. 'The Tangent or touch line is of great use 'in Trigonometry, insomuch that there is a Table 'made thereof, by which in several cases we are

‘forc’d to measure the Angles, not only of Plain, but also of Spherick Triangles, also in Opticks there are some propositions founded on touch Lines, particularly in determining the part of the Globe enlightened; the *Theory* of the Phases of the Moon is Established on this Doctrine; also that Celebrated Problem, by which *Hipparchus* findeth the Distance of the Sun by the difference of the true and apparent *Quadratures*. In Dialling the *Italian* and *Babylonian* hours are often Tangent lines, *Maurolicus* maketh use of touch lines, in finding the Diameter of the Earth; and in Navigation we take a Tangent line to be our Horizon.

Theorems of Proportion.

THEO. XV.

Magnitudes AB, and CD, that have the same proportion to a third E, F, have also the same one to another.

Dem. For A contains B as often as E doth F, (def. 48.) And C contains D as oft A : B as E doth F, therefore A contains B as 8 : 6 { E. F
oft as C doth D. A E C C:D { 10.5

Or thus $\frac{A}{B} = \frac{E}{F} = \frac{C}{D}$ 16:8

(def. 48.) therefore $\frac{A}{B} = \frac{C}{D}$ and therefore A:B::C:D. W.W.D.

Use. ‘This way of Arguing is often made use of, in several parts of the Mathematicks, particularly in the Solution of *oblique Spherical Triangles*.

THEO. XVI.

If there be several Proportional Magnitudes, viz. if

if $A : B :: C : D :: E : F$, then I say there will be the same reason of one Antecedent, to its Consequent, as of all the Antecedents, to all the Consequents. $A : B :: C : D :: E : F$. $9 : 3 :: 18 : 6 :: 24 : 8$

Dem. Since A is to B, as C to D, A will contain B, as oft as C contains D (*def. 48.*) Add both Antecedents together, as likewise both Consequents, then 'tis evident that A contains B, as oft as the Sum of A and C contains the Sum of B and D, therefore as $A : B :: A + C + E : B + D + F$. *W. W. D.*

Cor. Hence if like Proportionals be added to like Proportionals, the whole shall be proportional.

THEO. XVII.

What proportion any two Magnitudes as A and B. have one to another, the same proportion shall the like aliquot parts (suppose the fourth) of those Quantities, have one to another, that is as A to B, (the fourth of A) be to F, (the fourth of B.)

Dem. Divide A into 4 equal parts as E, H, I, K, and B into 4 equal parts, as F, L, M, N; Now I say, that as $E : F :: H : L ::$

$I : M :: K : N$ for all the antecedents are equal each to other, so likewise are all the consequents. But (*Theo. 16.*) as the Sum of all the Consequents,

so is one Antecedent to its Consequent, and therefore as $A : B :: E : F$. *W. W. D.*

Cor. Hence 'tis Evident, that the double, treble (or any other *Multiplex* whatsoever) of two Quantities have the same proportion as the Quantities themselves.

$$A \ 24 \left\{ \begin{array}{l} E \ 6 \\ H \ 6 \\ I \ 6 \\ K \ 6 \end{array} \right. B8 \left\{ \begin{array}{l} F \ 2 \\ L \ 2 \\ M \ 2 \\ N \ 2 \end{array} \right.$$

Use. 'These two Theorems do many times happen in the Mathematical Arguings; they are also necessary to demonstrate those that follow.

T H E O. XVIII.

If 4 Magnitudes are proportional, (viz if as $A : B :: C : D$) they shall also be alternately so, i. e. as $A : C :: B : D$.

Dem. If you deny it, and say there is a greater reason of A to C than of B to D, A will contain C more times than B doth D (*def.* 50.) let A contain C four times, and B contain D but three times, then the fourth of A will be equal to C, but a fourth of B will not be equal to D. Now seeing there is the same reason of $A : B :: C : D$, there will be the same reason of $\frac{1}{4} A : B$ as of $\frac{1}{4} C : D$, (*Theo.* 15.) and therefore if the $\frac{1}{4}$ of A be equal to C, the $\frac{1}{4}$ of B shall be equal to D, although the contrary follows if the former supposition be right, viz. that A hath a greater reason to C, than B to D, and therefore as A to C, so is B to D. *W.W.D.*

And here Note, That *Alternate* proportion hath place only, when the Quantities are of the same kind, for *Heterogenous* Quantities cannot be compared *Alternately*.

T H E O. XIX.

If there be 4 Magnitudes in the same proportion (viz. if $A : B :: C : D$) they shall also be proportional when inverted.

Dem. For if A be as great in respect of B, as C is

is in respect of D, then on the contrary B is as little in respect of A, as D is in respect of C, and therefore as
 $B : A :: D : C$ *W.W.D.*
 For the quantity of proportion is more generally defined by *how much fold*, rather than by *how many times* the consequent is contained in the antecedent.

If $A : B :: C : D$
 $6 : 2 :: 9 : 3$
 then inversely,
 $B : A :: D : C$
 $2 : 6 :: 3 : 9$

THEO. XX.

If Magnitudes Compounded be proportional, they shall be proportional when divided, that is if it be as $A : D :: B : F$ *it shall also be as* $A-D : D :: B-F : F$

Dem. Since there is the same Reason of A to D as of B to F, A will contain D so oft as B contains F; Now if D be taken from A and E from B once, or any other number of times, then (*ax. 3.*) D is contained in the Remainder A-D, as oft as F is in the Remainder B-F, and therefore (*def. 48.*) as $A-D : D :: B-F : F$ *W.W.D.*

Schol. Hence Compounded, Ratio is easily Demonstrated.

For if $A-D : D :: B-F : F$ make each Antecedent to contain his Consequent once more by adding the respective Consequents to them, then will $A : D :: B : F$ *W.W.D.*

Use. 'The foregoing Proportions very frequently occur in several Mathematical Argumentations.

THEO.

THEO. XXI.

If the whole be to the whole, as the part taken away is to the part taken away, the remainder shall be to the remainder, as the whole was to the whole, i. e. If it be as $A : B :: D : F$ it shall also be as $A-D : B-F :: A : B$.

Dem. Because $A : B :: D : F$ therefore alternately (*Theo.* 18.) 'twill be as $A :$

$D :: B : F$ and by Division If $A : D :: D : F$
'twill be as $A-D : D :: B-F : F$. $10 : 6 :: 5 : 3$

And again, by alternation as then 'twill be
 $A-D : B-F ::$ (*Hyp.*) $D : F :: A-D : B-F :: A : B$

$A : B$ therefore (*Theo.* 15.)

$A-D : B-F :: A : B$. *W.W.D.*

Cor. Hence if like Proportionals be taken from like Proportionals, the Remainder shall be Proportional.

Schol. Hence is converse If $A : B :: D : F$
Ratio demonstrated, for if $A :$ then by Conversion
 $B :: D : F$, then alternately as $A : A-B :: D : D-F$
as $A : D :: B-F$ but (*by the*
preceding Theo.) as $A : D :: A-B :: D-F$, and again
by alternation as $A : A-B :: D : D-F$. *W.W.D.*

After the like way of Arguing may mixt Ratio be demonstrated.

Use. 'This *Theo.* helps to demonstrate the Rule
'of Fellowship, for instead of working by the Rule
'of Three for every particular Associate, having done
'it for the rest, to the last they assign the Remain-
'der of the Gain; supposing that if there be the
'same proportion of the whole Sum of all the Prin-
'cipals to the whole Gain, as of the Principal of
'one Associate to his part of the Gain, there will
'also be the same proportion of the Principal that
'remains, to the remainder of the Gain. Some

Some other ways of arguing there are which sometimes are used, but these being the principal and most frequent, I omit them.

THEO. XXII.

having y^e same height

Triangles bhf , ehf are in such proportion to each other as their Bases bf , ef .

Dem. Let $bc = cd = de = ef$ draw bd , hc , then the Triangles bhc , ehc , dhe , ehf are equal (Theo. 8.) Now 'tis evident the Triangle bhf contains the Triangle ehf as oft as the base bf contains the base ef , and therefore (def. 48.) $\triangle bbf : \triangle ehf$, as the base $bf : ef$. W. W. D.



Schol. In like manner it may be proved, that pgrms. of the same or equal height have such proportion to each other as their Bases for (Theo. 17.) as the half is to the half, so is the whole to the whole.

Use. This Theo. shews how to cut off the third part of a Trapezium (as $abcd$) that hath two of its sides (as ad , bc) parallel.

Produce bc and make $ce = ad$, then take bg equal to the third of be and draw ag , I say that the Triangle abg is one third of the Trapezium $abcd$.

Dem. The Triangles adf , fce are equiangular because of the parallel Lines ad , be and vertical angles, there is also in each an equal side viz. ad , ce (Con.) thence they are equal to one another (Schol. Theo. 2.) and consequently the Triangle abe equal to the Trapezium $abcd$, but the Triangle abg is the third part of the Triangle abe by

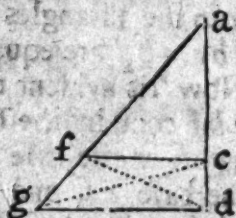


by this *Theo.* and therefore the Triangle abg is the third part of the *Trapezium* $abcd$.

THEO. XXIII.

Similliar or Equiangled Triangles afc , agd have their Corresponding sides proportional.

Dem. Make them coincident by laying the point a on a , and the right line ac on the right line ad &c. then I say fc is parallel to gd (Schol. 2. *Theo.* 1.) for the $\angle acf = \angle adg$. Draw the lines cg , fd , then the Triangles fcg , fdc are equal, being on the same base fc , and between the same parallels; But (*Theo.* 22.) $\triangle afc : \triangle fgc :: af : fg$, also $\triangle acf : \triangle cdf (= fgc) :: ac : cd$, and therefore (*Theo.* 15.) $af : fg :: ac : cd$, but (*Theo.* 18.) $af : ac :: fg : cd$. And (*Theo.* 16.) $af : ac :: af + fg (= ag) : ac + cd (= ad)$ and therefore again (*Theo.* 18.) $af : ag :: ac : ad$. W. W. D.



Schol. Hence if the sides of a Triangle be cut proportional, the right line that joins the points of Section, shall be parallel to the remaining side of the Triangle.

Use. This *Theo.* is of vast Extent, and may almost pass for an *Axiom* in all sorts of Measuring, it being the very foundation both of *Plain* and *Spherical Trigonometry*, for tho' spherical Angles are formed by Arches of Circles, yet must they all be reduced to Plain Triangles, before their parts can be measured: Its also from this Proposition they make and divide several Mathematical Instruments, as *Sinical Quadrant*, *Forestaff*, *Sector*, *Proportional Compasses*, with other Geometrical Instruments. In short,

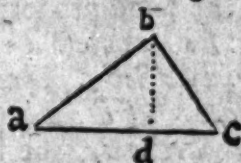
‘ short, ’tis so universal, that most of the usefulest
 ‘ parts of the *Mathematicks* are founded upon it.
 ‘ Its Uses and Applications being Infinite.

‘ Also upon this *Theo.* depends the practice of that
 ‘ Problem, which teaches how to cut a right line in-
 ‘ to any number of equal parts, See the manner of
 ‘ doing it *Prob.* 10.

T H E O. XXIV.

*A Perpendicular let fall from the Right Angle of
 a right angled Triangle a b c, divideth it into two Si-
 millar Triangles a d b, b d c, which are also simi-
 lar to the whole.*

Dem. The $\angle a b c = a d b$ because both right,
 and the $\angle a$ common, therefore
 the Triangles $a b c$, $a d b$ are fi-
 millar (*Theo.* 4. *Cor.* 3.) By the
 same Argument are the Trian-
 gles $a b c$, $b d c$ fimillar, and
 therefore (*ax.* 1.) the Triangles $a d b$, $b d c$ are
 fimillar. *W. W. D.*



Use. ‘ This *Theo.* is often used in solving Geo-
 ‘ metrical Questions by *Algebra*; Also by finding
 ‘ the sides of the five Regular Platonick Bodies
 ‘ inscrib’d in a Sphere. Its also by this *Theo.* and
 ‘ the help of a square, an inaccessible distance as $d a$
 ‘ may be measured, for draw the perpendicular $b d$,
 ‘ and set a square at the point b , so that by looking
 ‘ over the sides $b a$, $b c$ you may see the points a
 ‘ and c . Now because by this *Theo.* $c d : d b :: d b$
 ‘ $: d a$, if therefore you multiply $d b$ by it self, and
 ‘ divide the Product by $c d$, the Quotient shall be
 ‘ $d a$ the Line required.

T H E O.

THEO. XXV.

Equal Equiangled pgrms. as a d, d g have their sides which are about the equal Angles reciprocal, And equiangular pgrms. that have those sides reciprocal are equal.

Dem. Let the sides $c d$, $d e$ about the equal Angles make one right line, wherefore $f d$, $d b$ (Theo. 1. Cor. 2.) shall do the same, and let $a b$, $g e$, be produced (Post. 1.) till they meet in h .

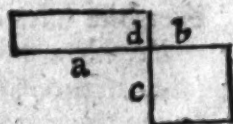


First therefore the pgrms. $a d$, $d g$ being equal have the same proportion to the pgrm. $b e$, but pgrm. $a d : b e ::$ base $c d : d e$, and pgrm. $d g : b e ::$ base $f d : d b$ (Schol. Theo. 22.) therefore (Theo. 15.) $c d : d e :: f d : d b$. W. W. D.

Secondly, the sides of the pgrm. being reciprocal there is the same proportion of the base $c d : d e :: f d : d b$, but as $a d : b e ::$ base $c d : d e$ (Theo. 22.) $:: f d : d b$ (Hyp.) $:: d g : b e$ (Theo. 21.) therefore (Theo. 15.) pgrm. $d g : e b$, and therefore pgrm. $a d = d g$. W. W. D.

THEO. XXVI.

If four lines are proportional, suppose $a : b :: c : d$, the rectangle of the first and fourth, shall be equal to the rectangle of the second and third. And contrary, If the rectangle of the Extremes and Means are equal, the four Lines are proportional.



Dem.

Dem. The Rectangles have each an equal Angle because both right, and $a : b :: c : d$ (*Hyp.*) therefore by the last part of (*Theo.* 25.) the Rectangles are equal.

Secondly, If they are equal and equiangled, their sides will be reciprocal, (by the first part of *Theo.* 25.) that is, as $a : b :: c : d$. *W. W. D.*

Schol. In like manner it might be proved, that if three lines are proportional, the square of the middle term shall be equal to the Rectangle of the first and third.

Use. 'By these two Propositions is demonstrated
' the Principal Rules of Common *Arithmetick*, viz.
' The *Golden Rule*, commonly call'd the *Rule of*
' *Three*, the *Rules of Fellowship*, *Alligation*, with
' others that depend upon the Doctrine of Proportion.
' For suppose it be required to find
' a fourth Proportional to the three Num- a, b, c .
' bers a, b, c . Then because by this Pro- $3, 9, 6$.
' position $9 \times 6 = 3$ multiplied by ano-
' ther Number unknown. If then I divide 54 by 3
' the Quotient is 18, which if multiplied by 3 will
' give $54 = 9 \times 6$, and therefore if 3 give 9, 6
' shall give 18 for the fourth proportional required.

THEO. XXVII.

Equal triangles acb, ecd that have in each an equal Angle, viz. acb, ecd have the sides (ac, cb and ec, cd) which form the equal Angles reciprocal, And contrary, if the Angles are equal and sides reciprocal, the triangles shall be equal.



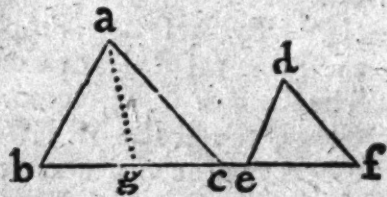
Dem. Let the sides ac, cd which are about the equal Angles be set in a straight line, then, bce is a straight line (*Theo.* 1. *Cor.* 2.) join ae . *First*,

First, As $ac : cd :: \triangle aec : \triangle ced$ (Theo. 22.)
 $:: \triangle eac : \triangle cab$ (because equal) $:: ec : cb$ (Theo.
 22.) and therefore as $ac : cd :: ec : cb$. W.W.D.

Secondly, $\triangle aec : \triangle ced :: ac : cd$ (Theo. 22.)
 $:: ec : cb$ (Hip.) $:: \triangle eca : \triangle cba$ (Theo. 22.)
 therefore (Theo. 15.) $\triangle aec : \triangle ced :: \triangle eca :$
 $\triangle cba$, and therefore (def. 48.) $\triangle ced = \triangle cba$.
 W.W.D.

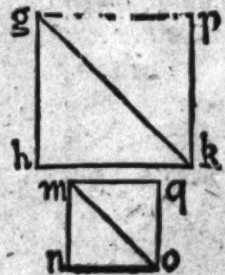
THEO. XXVIII.

Simillar triangles
 abc, def , are in a
 duplicate Proportion of
 their Homologous sides
 that is, their Area's
 are as the squares of
 their Homologous sides.



Dem. Let $bc : ef :: ef : bg$ and let ag be drawn.
 Now because $ab : de :: bc : ef ::$ Theo. 22.) $ef : bg$
 (Con.) and the Angle $b = e$, therefore is the Tri-
 angle $abg = def$ (Theo. 27.) but $\triangle abc : \triangle abg$
 $:: bc : bg$ (Theo. 22.) Now the proportion of bc
 to bg is duplicate to that of bc to ef (def. 55.)
 therefore $\triangle abc$ is to $\triangle abg (= def)$ in dupli-
 cate ratio to that of bc to ef , that is Triangle $\frac{abc}{def} = \frac{bc}{ef}$
 twice. W.W.D.

Secondly, that this proportion
 is as the squares of their Homo-
 logous sides I thus prove, Let the
 Triangles be ghk, mno (grant-
 ing all Triangles this property)
 which severally are half of the
 Squares bp, nq . Now since the
 half is to the half as the whole



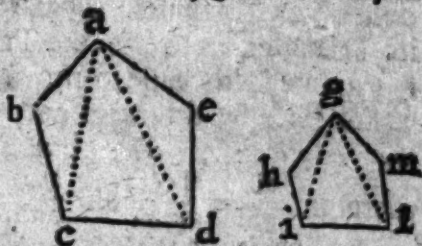
to the whole, the Triangle gbk is to the Triangle mno , as the square of bk to the square of no . *W.W.D.*

Schol. Hence 'tis easie to prove that all similar superficies are in a duplicate proportion of their Homologous sides, or as the squares of their Homologous sides.

Before I pass to the proof of this Excellent Theorem, it's necessary to premise the following Lemma.

Lem. Similar Polygons $abcde$, $ghilm$ may be

divided into an equal Number of similar Triangles, which Triangles will be like parts of their respective wholes. *First*, As



to their Number 'tis evident by Inspection. *Secondly*, That they are like I thus prove, the $\angle b = \angle h$ (*Hyp.*) and the side $ab:bc::gh:hi$ (*Hyp.*) and therefore if the Triangles are made coincident the line gi will be parallel to ac (*Schol. Theo. 23.*) and consequently the Triangles abc , ghi are similar.

By a like Argument are the Triangles aed , gml similar. *Lastly*, the $\triangle cad$ like igl , for the whole Figures were equiangled, and equal Angles have been taken away on each side: Therefore the Triangle cad is similar to igl . *Thirdly*, That they are like

parts of their respective wholes. Triangle $\frac{abc}{ghi} = \frac{ab}{gh}$ twice (by the preceding *Theo.*) so also $\frac{acd}{gil} = \frac{cd}{il}$

twice, and $\frac{aed}{gml} = \frac{ed}{ml}$ twice, and because $ab:gh::$

E

e d

$cd:il::ed:ml$, therefore $\frac{ab}{gh} = \frac{cd}{il} = \frac{ed}{ml}$ and therefore Triangle $\frac{abd}{ghi} = \frac{aed}{gml} = \frac{acd}{gil}$ that is Triangle $abc:ghi::aed:gml::acd:gil$. But (Theo. 16.) $\triangle abc:\triangle ghi::abc+aed+acd:ghi+gml+gil$, and (Theo. 18.) $\triangle abc:abc+ead+acd::\triangle ghi:ghi+gml+gil$, And again, $abc+aed+acd:\triangle abc::ghi+gml+gil:\triangle ghi$, that is, the Polygon $abcde$ is to any one of its Triangles, suppose abc , as the other of its Triangles, as to the like Triangle ghi . This being granted, 'tis easie to prove what was proposed.

For since by the third part of the preceding Lemma $\triangle abc:\triangle ghi::Pol. abcde:Pol. ghilm$, therefore $\frac{abc}{ghi} = \frac{abcde}{ghilm}$ but $\frac{abc}{ghi} = \frac{ab}{gh}$ twice, therefore (ax. 1.) $\frac{abcde}{ghilm} = \frac{ab}{gh}$ twice, And therefore $Pol. abcde:Pol. ghilm$ is in duplicate Ratio of $ab:gh$, or as the square of ab to the square of gh . *W.W.D.*

Cor. Hence Circles are in a duplicate Ratio to that of their Diameters; for a Circle may be divided into an infinite number of Triangles, as was intimated in (Theo. 8.)

2. Hence if three right lines are proportional, then as the first is to the third, so is a Polygon made upon the first, to a Polygon made upon the second, like and alike described, or so is a Polygon made upon the second to a Polygon like, and alike described on the third.

3. If the *Homologous sides* of like Figures be known, then will the proportion of the Figures be evident, viz. by finding a third proportional.

Use. Hence is discover'd a method of enlarging or

' or diminishing any right lin'd Figure in a given Ra-
 ' tio: As if you would make a pentagonal Polygon
 ' six times as big as another, one of whose sides is
 ' $a b$, then betwixt $a b$ and 6 times $a b$ find a mean
 ' proportion, upon which describe a Polygon like the
 ' former, and it shall be *Sextuple* of the *Hexagon*
 ' given, as is evident from the preceding Demonstra-
 ' tion. The contrary of this is also manifest, as sup-
 ' pose the like sides of two simillar Polygons, and
 ' the Area of one of them be given to find the Area
 ' of the other, then because their Area's are as the
 ' squares of their like sides, it will hold as the square
 ' of the side of the Polygon, whose content is given,
 ' to the square of the other side, so is the con-
 ' tent of the given Polygon, to the content of
 ' the Polygon required. As, suppose 512 be the
 ' Area of a superficial Figure, one of whose sides is
 ' 24, what shall be the Homologous side of another
 ' like Figure whose Area is 768, say as 512 to 768,
 ' so is 24 to 36, which gives the proportion of their
 ' Area's, whose first term is the given side, then be-
 ' twixt 24 and 36 find a mean proportional, that is,
 ' extract the square Root of 24 by 36 it gives the
 ' side required.

Lastly, ' It corrects the Error of those who are
 ' apt to conclude that *Polygons* are in such Propor-
 ' tion to each other as their sides; For if two *Trian-*
 ' gles, two *Squares*, two *Polygons*, &c. are propo-
 ' sed, and the side of the first double to to the side
 ' of the second, then shall the first Figure be four
 ' times as big as the second.

T H E O. XXIX.

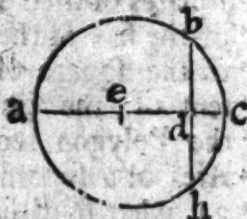
If two right lines $a b$, $c d$ intersect one another in
 a Circle, the Rectangle made of the parts of the one,

is equal to the rectangle made of the parts of the other, that is, $e a \times e b = d e \times e c$.

Dem. Join the points $d b, a d, c b$, The $\angle c e b = \angle a e d$ because Vertical, also the $\angle a = \angle c$ (Cor. Theo. 12.) and consequently the $\angle a d e = \angle c b e$, therefore the Triangles $a e d, c e b$ are simillar, and therefore (Theo. 23.) $a e :: e c :: d e :: e b$ wherefore (Theo. 26.) $e a \times e b = d e \times e c$. W.W.D.



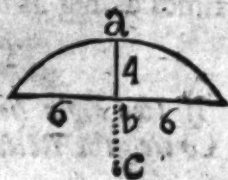
Schol. Hence is gotten a method of finding a mean proportional betwixt two given lines; Suppose $a d$ and $d c$. In an infinite right Line, as $a c$, assume the point d , and on each side it (post. 3.) cut off $d a$ and $d c$, and let through d the right Line $b b$ be drawn at right Angles to $a c$, bisect $a c$ in e , and describe with the distance $e a$ or $e c$, the Circle $a b c$, then shall the line $d b$, or $d b$ cut off by the periphery, be the mean proportional required, it is also the side of a Square equal to a Rectangle made of the Lines $a d, d c$.



Dem. For $d b = d b$ (Theo. 10.) and by the preceding Theo. $b d \times d b = a d \times d c$, that is $b d q = a d c$ and therefore (Theo. 26.) as $a d :: d b :: d b :: d c$. W.W.D.

Use. This Theo. gives Artificers, an Arithmetick Rule, for finding the Center to strike an Arch to any Height and Breadth; suppose four Foot high, and twelve Foot broad.

Rule, Square half the breadth which is 6, it give 36, divide this by 4, it gives 9, which 9 added to 4 the height, gives 13; half of which is $6\frac{1}{2}$, this



set

set from the top of the Arch *a*, upon the Line *a b* produced, will give you *c* the Center, upon which you may strike it to the given Dimensions; see the Figure:

THEO. XXX.

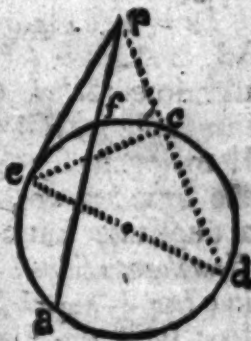
If from a point *p* without a Circle, you draw two right Lines *p a*, *p e*, to the opposite part of the periphery; the Rectangle of one whole Line *p a*, with that part of it *p f*, cut off without the Periphery, shall be equal to the rectangle of the other whole Line *p e*, and that part of it *p d*, cut off as before.

The Triangles *p a e*, *p d f*, are simillar for the four sided Figure *a f d e* being inscribed, in a Circle makes the Angle $p f d = p e a$, (they being both the Complement of the Angle *a f d*, to a Semicircle,) and the Angle *p* is common, therefore are the Triangles *p f d*, *p e a* Simillar, and Consequently have their corresponding sides proportional, wherefore as $a p : e p :: d p : f p$ therefore (Theo. 26.) $a p \times f p = e p \times d p$. VVVVD.

Schol. Hence 'tis easie to prove, that if from a point *p* without a Circle, two right lines as *p e*, *p a*, be drawn, so that one *p a* may intersect the Circle, and the other *p e* touch it; the Rectangle made of the one whole line *p a*, with that part of it *p f* cut off without the Periphery, shall be equal to the square of the tangent line *p e*.

E 3

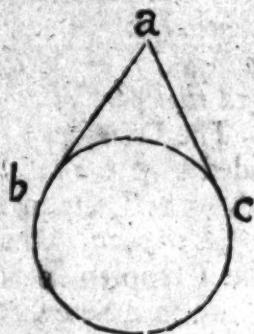
Dem.



Dem. From e draw the Diameter ed , join the points pd as also ec , Then are the Triangles pce , pde fimillar; For the Angle at p is common, also the Angle pde is right (*Theo.* 14.) and therefore equal Angle ecd because in a Semicircle, consequently the remaining $\angle pec = pde$, and therefore $pc : pe :: pe : pd$, so that $pc \times pd = peq$. But by the preceding *Theo.* $pc \times pd = pt \times pa$, therefore (*ax.* 1.) $pt \times pa = peq$. *W.W.D.*

Cor. Hence it appears that two tangents as ab , ac drawn from the same point without the Circle are equal.

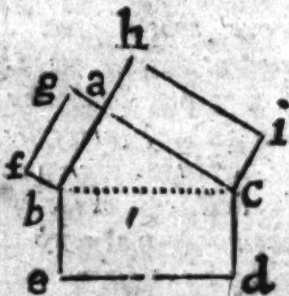
Use. 'The principal use of this *Prop.* is to demonstrate the third and fourth Axioms in plain Trigonometry.



THEO. XXXI.

In a right angled triangle bac any Figure as bd , describ'd on the side bc subtending the Right Angle, is equal to both the Figures bg , ai described on the sides ba , ac containing the right Angle, like and alike situate to the former bd .

Dem. The Figures fa , ai , bd , are alike (*Hyp.*) and therefore (*Schol. Theo.* 28.) in a duplicate Ratio of their Homologous sides; Also by the same baq , acq , bcq are in a duplicate Ratio of their Homologous sides; Now seeing $baq + acq = bcq$, therefore shall the Figure $fa + ai = bd$ (*Theo.* 15.) *W.W.D.*



Schol.

Schol 1. Hence a Circle describ'd upon the Hypotenuse of a right angled triangle, is equal to the two Circles describ'd upon the other two sides of the triangle; For (Schol. Theo. 28.) Circles are also in a duplicate Ratio to that of their Diameters.

Note, Such part as eb is of bc , such part must ah be of hi , and such part must ag be of gf . See (def. 35.)

Schol. 2. Hence may be learnt how to Add or Substrail any like Figure, viz. by using the same method that was follow'd in adding and substracting of squares in Theo. 9.

Use. ' This Theo. is exceeding useful, it being an ' universal Application of the 9 Theo. to all manner ' of Figures.

Of Solids.

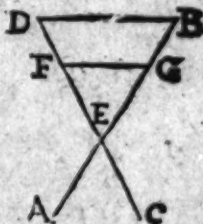
THEO. XXXII.

One part AC of a right line cannot be in a plain superficies, and another part CB out of it.

Dem. Produce AC to E , now if you conceive CB to be drawn straight from A C , then two right lines ACB , ACE have one common Segment AC , Which (ax. 8.) is Absurd.



Schol. 1. Hence any triangles as DEB is in one and the same plain. For imagine EFG part of the Triangle DEB to be in another plain, then EF part of the right line ED is in one plain, and the other part FD elevated above it, Which is Absurd, by the preceding Theo.



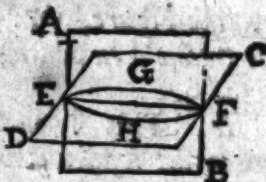
Schol. 2. Hence also if two right lines as AB , DC in the last Fig. cut one another, 'tis evident they are in the same plain.

Use. 'This Proposition is of use in *Dyalling*, for
'by it we prove, that the *shade* of the *Style* cannot
'fall out of the plain of that great Circle, in which
'the Sun is.

THEO. XXXIII.

If two Plains *AB*, *CD* cut one the other, their
common Section *EF* is a straight line.

Dem. If *EF* be not a straight
line, then draw the straight
line *EGF* in the Plain *AB*,
and the straight line *EHF* in
the Plain *CD*: Then 'tis evi-
dent two straight lines *EGF*,
EHF include a Superficies, Which (*ax. 9.*) is Absurd.

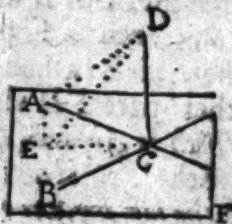


Use. 'This Proposition is of great use in several
'parts of the *Mathematicks*, but particularly in *Dy-*
'*alling*, for the Hour lines on all Plains are only the
'common Section of the Hour Circles Plain, with
'that of the Wall.

THEO. XXXIV.

A straight Line *CD* that is perpendicular to two
lines *AC*, *BC* crossing, is also perpendicular to their
Plain *AF*.

Dem. If you deny it, let *DE* be perpendicular to
the plain *AF*, join the points *EC*, as also *DA*, and
at right Angles to *EC* draw *EA*: Now *DE* is per-
pendicular to *EC* and *EA*
(*Theo. 32. Schol. 1. def. 68.*)
also *DC* perpendicular to *CA*
(*Hyp.*) therefore *ADq = ACq*
 $+ DCq$ (*Theo. 9.*) and *ACq*
 $= AEq + ECq$, also *DCq =*
 $EDq + ECq$ (*Theo. 9.*) and



there-

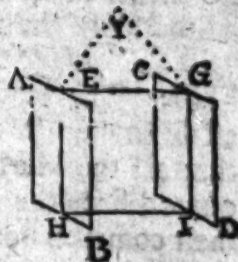
therefore $ADq \dot{=} AFq + EDq$ (ax. 5.) and consequently Angle AED cannot be right. The same reason will hold, if the line DE touches the plain any where else but in C , and so not DE but DC is perpendicular to AF the plain. *W.W.D.*

Use. 'Theodosius demonstrates by this proposition, that the Axis of the World is perpendicular to the plain of the Equator; It's likewise used in Dyalling, also in Astralabes, and in cutting of Stones.

THEO. XXXV.

If a plain HG cut two parallel plains AB, CD , its common Sections EH, GF with them shall be parallel lines.

Dem. For if they be otherwise, being in the same plain, they will if produced meet somewhere, suppose in I ; Now since the whole lines HEI, FGI are (Theo. 33.) in the plains AB, CD therefore these plains being produced, shall also meet, Which is contrary to the Hypothesis.



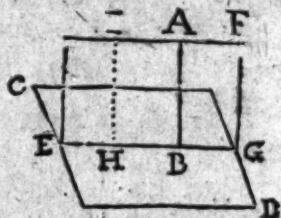
Use. 'This Proposition is of greater use in Conick-Sections, for by it we prove that if the Cone or Cylinder be cut by a plain parallel to its base, the Sections are Circles; In Dyalling we prove, that the Angles the Hour Circles make with a plain parallel to a great Circle, are equal to those which they make with the Circle it self, likewise in Perspective 'tis proved by this Theo. that the Image of the objective lines which are perpendicular to the Table, concur in the point of Vision.

THEO.

THEO. XXXVI.

If a Right Line AB be perpendicular to some plain CD , all the plains in which that right line is, shall be perpendicular to the said plain CD .

Dem. Let some plain as EF be drawn by the line AB , making the Section EG with the plain CD ; from some point as H let be drawn HI parallel to BA in the plain EF , then shall HI be perpendicular to the plain CD , because the straight line HG makes equal Angles with the parallels IH , AB , and therefore the plain EF is perpendicular to the plain CD . *W. W. D.*



Use. The very fundamentals of all Dyalling is 'establish'd on this Proposition, for by it we prove 'that all the Meridians are perpendicular to the Equator, since the Axis of the World is so, which is 'their common Section; 'tis likewise of great use in 'Spherick Trigonometry, Perspective, and all other 'parts in which we are obliged to consider plains.

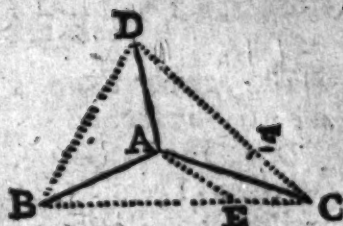
THEO. XXXVII.

If a solid Angle $ABCD$ be contained under three plain Angles BAD , DAC , BAC any two of them taken together, are greater than the third.

Dem. If the three Angles are equal, the affection is evident, if unequal let the greatest be BAC , from whence take away $BAE = BAD$ and make $AD = AE$ drawing BE , BD , DC . Now the side BA is common, and $AD = AE$ (*Con.*) Also $BD + DC < BC$ (*def. 2.*) therefore rejecting BD and BE in both, there shall remain $DC < EC$.

But

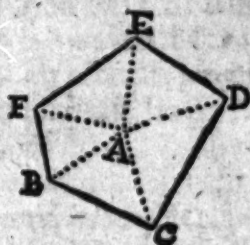
But since $AD = AE$
and AC common, also
 $DC = FC$ the Angle
 CAD shall be greater
than EAC (for if E be
turned over AC it will
fall but at F) Therefore
the angle $BAD + CAD = BAC$. *W. W. D.*



THEO. XXXVIII.

Every Solid Angle A is contained under less than 4 right Angles.

Dem. Let a plain cut off the top of the solid Angle, and make the many sided Figure $BCDEF$, which from some point in the Figure let be cut into Triangles, then let the Sum of the Angles B, C, D, E, F , of the said Figure be denoted by x , and let y represent the Sum of all the Angles at the bases of the Triangles which form the solid Angle; Then $x + 4$ right Angles $= T + A$ (*Theo. 4.*) for $T + A = 10$ right Angles, and so is $x + 4$ right. But the $\angle ABC + \angle ABF = \angle CBF$ (*Theo. 37.*) the same holds in the Angles C, D, E, F . And therefore 'tis evident that $T < x$, and consequently that A which denotes the solid Angles, is less than 4 right. *W. W. D.*



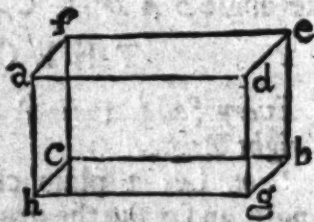
Use. These two Propositions are made use of to demonstrate some that follow, as also when of several plain Angles there may be made a solid Angle, the knowledge of which is very requisite, in the cutting several Bodies in Wood, Stone, &c.

THE Q.

THEO. XXXIX.

The opposite Plains ac , db of a Parallelepipedon are alike and equal.

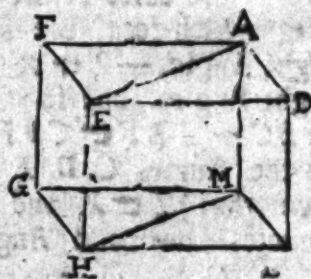
Dem. The plains ac , db are parallel (*def.* 89.) therefore (*Theo.* 35.) the interfections af , de are parallel as are also ef , da and therefore ae is a *pgrm.* and by Consequence $af = de$. In like manner it might be prov'd that all the other plains are *parallelograms*, and that the sides in one *pgrm.* are respectively equal to the sides in its opposite: Therefore the Angle $fab = edg$ and $afc = deb$ and therefore the plain ac is equal and like to the plain db . *W.W.D.*



THEO. XL.

A Parallelepipedon FL is divided into two equal Parts, by the Diagonal plain HA .

Dem. All the opposite Parallelograms are equal (*Theo.* 39.) And the Triangle $AED = AEF$, also the Triangle $LMH = MHG$ (*Theo.* 6.) and the plain AH is common, therefore the solid $AGH = HDA$. *W.W.D.*

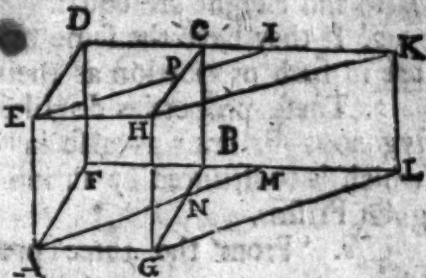


THEO. XLI.

Parallelepipedons AD and CK are equal, which have the same or an equal base, and the same or an equal height.

Dem.

Dem. The Prisms $AFMEDI$, $GBLHCK$ are equal (*def.* 74.) from both these take away the common Prism NBM PCI , and add to the remainders the solid $AGNEHP$ it makes the Parallelepipedon ACE equal to the Parallelepipedon $CK.W.W.D.$

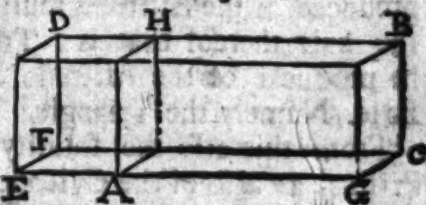


This Proposition may more universally be prov'd (that is for all sorts of Prisms) by Infinities after the method of (*Theo.* 7.)

THEO. XLII.

Parallelepipedons AD, AB of the same or equal height, are one to another as their Bases.

Dem. Let the Base AF be conceiv'd produced to GC , and the plain AH which is parallel to ED (*def.*



89.) be mov'd parallelly to GB , it shall at the same time sweep the Base AC and the Parallelepipedon AB , and by consequence how much soever it has taken away at any time from the Base AC , it will have taken away alike part from the Parallelepipedon AB ; and therefore as the Parallelepipedon AD to the Parallelepipedon AB , so is base AF to base AC . *W.W.D.*

What has been demonstrated of Parallelepipedons in the 41 and 42 Propositions, holds also in triangular Prisms, they being half thereof, by (*Theo.* 40.) And therefore

Cor.

Cor. 1. Triangular Prisms which have the same Base and Height are equal.

2. If they have the same or an equal height, they are in such proportion as their Bases.

3. These properties do likewise hold in all polygonous Prisms, as also in the Cylinder it self, which is constituted by an infinite number of Triangular Prisms.

Use. From the three preceding Theorems is learnt the way of measuring all sorts of Prisms, whether Triangular, Quadrangular or Multangular, viz. by multiplying the Area of the Base into the Altitude. As suppose the Area of the Base of a right angled Parallelepipedon be 48 foot, and the Altitude 17 foot, then multiplying 48 by 17, the product 816 is the solid Content of the said Figure in Cubick Feet.

Moreover, Since the whole Parallelepipedon is produced of the Altitude multiplied into the Base, the half thereof that is a Triangular Prism shall be produc'd of the Altitude drawn into half the Base. Namely the Triangle.

From this also the solidity of all polygonous Prisms, as also of the Cylinder is found, viz. by multiplying the Area of the Base into the height.

For, as a Rectangle is produced by multiplying the length of the Base into the height, so is the content of a right Parrallelepipedon and all Prisms produc'd, by multiplying the Area of the base by the Altitude; And therefore (*Theo. 41.*) the solidity of all Prisms is found by multiplying the base by the height.

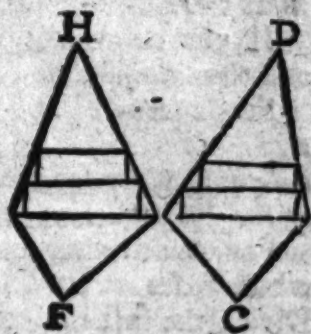
THEO. XLIII.

All Pyramids as C D, F H having the same Altitude and equal Bases are equal.

Dem.

Dem. If you deny it let CD be greater than FH , and let there be inscrib'd in this Pyramid any number of Prisms having like Bases and equal Heights, whose solidity may be greater than the Pyramid FH ; then these Prisms so inscrib'd, will be less than the Pyramid in which they are inscrib'd; for the lesser heights the Prisms have, the more will there be in number, *that is*, by lessening their height you may encrease their number, so

that it shall come nearer to the Pyramid CD , than any other Quantity, suppose FH . Let there be also inscrib'd in the Pyramid FH the same number of Prisms as was in the Pyramid CD . I say the Prisms in each of these Pyramids are equal, for



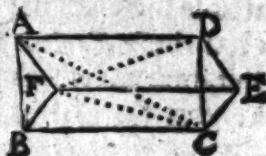
their number is equal by *Construction*, their Bases and Heights may be also equal, because those of the Pyramids in which they are inscrib'd, are so (*Hyp.*) *that is*, all the Prisms in $H F$ is equal to all the Prisms in CD , which by construction was made greater than the Pyramid FH , *that is*, a part is equal to the whole, Which is impossible; And therefore the Pyramid $CD = FH$. W.W.D.

Schol. Hence all Cones are equal, which have an equal height and base; for they may be divided after the same manner, and therefore the like Argument will hold.

T H E O. XLIV.

Every Prism as $ABCDEF$ having a Triangle for its Base, may be divided into three equal Pyramids $FDEC$, $CBFA$, $FADC$. *Dem.*

Dem. Draw Diagonals FC , FD , AC which shall biseft the *pyram* (*Theo. 6.*) Now the Pyramid FDC = *pyram.* $CBFA$ because they have equal bases and heights; Also for the same reason *pyram.* $FACB$ = *pyram.* $FADC$, therefore (*ax. 1.*) the three Pyramids $FDEC$, $CBFA$, $FADC$ are equal *W. W. D.*



If you cut this Figure out in Cork, or the like, and then consider it, the *Dem.* will be very clear.

Schol. Hence 'tis easie to prove that every Pyramid is a third part of the Prism, that has the same base and Altitude with it; Or every Prism is treble of the Pyramid that has the same base and Altitude.

For conceive any polygonious Prism and Pyramid whose bases are equal and like and height the same; Divide the first into triangular Prisms, and the other into triangular Pyramids, then all the parts of the Prism shall be treble to all the parts of the Pyramid, by the preceding *Theo.* and consequently the whole Prism is treble to the whole Pyramid. *W. W. D.*

Schol. 2. Hence also we may reasonably conclude, that a Cone is the third part of a Cylinder, that has the same Base and Altitude with it. For the Cylinder may be divided into an infinite number of triangular Prisms, and the Cone into a like infinite number of triangular Pyramids.

Schol. 3. Hence, and from the Corollaries (of Theo. 42.) Pyramids of the same height are in such proportion as their Bases;

For by those Corollaries all Prisms are so. Now Pyramids of the same base and height are the third of those Prisms, by the preceding *Theo.* therefore in such proportion as their bases; For as the whole is

to

to the whole, so is any part to the like part. This property will also hold in Cones.

Use. 'Upon this *Theorem*. is founded the reason of measuring all Pyramids, whether Triangular, Rectangular or Multangular, as also of the Cone or Sphere; for since (*Schol.* 1.) of the preceding *Theo.* every Pyramid is $\frac{1}{3}$ of the Prism that has the same base and height with it, therefore the solidity of all Pyramids, is found by multiplying the Area of the Base by $\frac{1}{3}$ of the Altitude, for that gives $\frac{1}{3}$ of the solidity of the Prism, which (by the preceding *Theo.*) is equal to the Pyramid of the same base and height; The same Rule holds also in Measuring the Cone, that being the $\frac{1}{3}$ of the Cylinder of the same base and height with it.

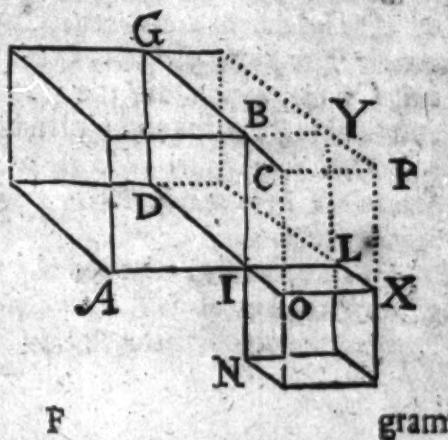
Lastly, 'The Sphere (which is composed of an infinite number of Pyramids, the Sum of whose bases is the whole surface of the Sphere, and the height the Radius of the Sphere) is measured by multiplying the Area of the surface by $\frac{1}{3}$ of its Semi diameter.

THEO. XLV.

Similar Parallelepipedons A G, N X are in a triplicate proportion of their Homologous sides, that is, their solidities are as the Cubes of their Homologous sides.

Dem. Let the Parallelepipedons be A G, N X.

Fill up the void space with the Parallelepipedons D Y, I P, then shall the side A I: I L:: D I: I O: B I: I N (*Hyp.*) also Parallelo-



gram $AD : DL :: DL : IX :: BO : IT :$ (*Schol. Theo. 22.*) that is, the Parallelepipedon $AG : DY :: DY : IP :: IP : NX$ (*Theo. 42.*) therefore the proportion of $AG : NX$ is triple of $AG : DY$ that is, of $AI : IL$ their Homologous sides. *W.W.D.*

Secondly, That this Proportion is as the Cubes of their Homologous sides I thus prove, All Cubes are like Parallelepipedons, and consequently are in a triplicate proportion of their Homologous sides by the preceding *Theo.* But like Parallelepipedons are in the same reason as Cubes; for upon two given lines conceive two simillar Parallelepipedons made, and upon the same lines two Cubes, then because the Parallelepipedons are in a triplicate Ratio of those given lines, and also the Cubes in a triplicate Ratio of the same Lines, therefore the Parallelepipedons are one to another as the Cubes of their Homologous sides.

Cor. 1. What has been prov'd in the preceding *Theo.* concerning like Parallelepipedons, holds also in like triangular Prisms as being their halves (*Theo. 40.*) As also in all Polygonous Prisms, of which the Triangular is an Aliquot part. And consequently in the Cylinder, which is made up of an infinite number of triangular Prisms. So that the solidity of all Cylinders are as the Cubes of their Diameters. The same property holds also in all Pyramids and Cones, which are the third part of their Circumscribing Prisms and Cylinders.

Schol. Hence 'tis easie to prove that all simillar Solids are in a triplicate proportion of their Homologous sides.

All like Solids, whether they be Regular or Irregular, may be resolv'd into simillar Pyramids: For let there be two simillar Solids, in one of which conceive

ceive a point, from which Imagine straight lines drawn to all the Angles of one of the Figures, divided into Pyramids, whose number are equal to the number of Plains that bound the solid; do the like in the other solid, *that is*, conceive a point posited in all respects here as it was in the other, *that is*, so that the sides of any one of these Pyramids in one of these Solids; be proportional to the like sides of the like Pyramid in the other solid, then 'tis naturally evident that these two solids are divided into an equal number of simillar Pyramids, which are like parts of their respective wholes. Now since these Pyramids (which are like aliquot parts) are in a triplicate proportion of their sides, and these aliquot parts in such proportion as their wholes (*Cor. Theo. 17.*) therefore the whole solids must be to each other in a triplicate proportion of their Homologous sides.

Cor. Hence 'tis evident that Spheres are in triplicate proportion of that of their Diameters; For a Sphere is composed of an infinite number of like Pyramids, whose bases are part of the surface of the Sphere, tops the Center of the Sphere, and heights the Radius of the Sphere. Now these Pyramids are to each other as the Cubes of their heights, which is the Semidiameter of the Sphere, and which I take for one of their sides, as differing insensible therefrom; But these Pyramids being like parts of their wholes, therefore the whole Spheres are in a triplicate proportion of that of their Semidiameters: Now the Semidiameters are to each other as their whole Diameters; therefore Spheres are in a triplicate proportion of that of their Diameters.

Use. 'Upon this *Theorem* is grounded many excellent Properties, particularly the augmenting or encreasing

' encreasing Solids in a given Ratio, as suppose I
 ' have a Cube whose side is 7 Inches, and I would
 ' find the side of another Cube that is double to it :
 ' Here 'tis evident that the side of this second Cube
 ' cannot be double the side of the first, for if so,
 ' then like Solids were in such proportions as their
 ' sides, which the preceding *Theo.* proves otherwise,
 ' that is, that they are in a triplicate Ratio of their
 ' Homologous sides; And therefore, to find to the
 ' side of this double Cube, 'tis only to find two mean
 ' proportionals betwixt the side of the Cube propos'd
 ' and double its side; Hence 'tis easie to solve that
 ' Celebrated Proposition of the duplication of the
 ' Cube propos'd by the Oracle.

' Also by this Proposition we correct the false o-
 ' pinion of those which imagine that like Solids are
 ' in the same Ratio as are their sides, *that is*, that a
 ' Cube whose side is a foot, is but half of the solid
 ' whose side is two foot. Also in Naval Archi-
 ' tecture, if you would keep or observe the same
 ' proportion: For Example, If the Keel of a Ship
 ' of 150 Tun be 54 foot long, what length ought
 ' the Keel of that Ship to be whose Burthen is
 ' 350 Tun. Say first, if a 150 give 360, what shall
 ' 54 give, the answer is 126, the proportion of the
 ' Solidities, whose first term is the given side; Find
 ' therefore two mean proportionals betwixt 54 and
 ' 126, by extracting the Cube Root of the Product
 ' of the Square of 54 into 126, *that is*, of 367416,
 ' and it gives you 72 *prope* the Keel of the Ship
 ' required; Having thus gotten the length of the
 ' Keel of a Ship of this Burthen, the Dimensions of
 ' all the other Members that compose the Ship are
 ' easily acquir'd by a single Proportion thus; As
 ' 54 foot to 72 foot, so is 36 foot the length of a
 Beam

' Beam in the first, to 48 the length of the Beam in
' the second ; For 'tis supposed that the Dimensions
' of all the Members and Parts of any one Vessel
' be known.

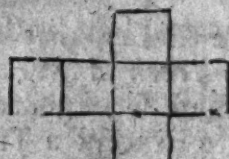
THEO. XLVI.

*There can be but 5 Regular Bodies, that is, such
that have all their sides, and all their Angles equal.*

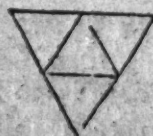
Dem. There is no plain Figure when join'd together can make a solid Angle, except a Triangle Square and Pentagon : For a solid Angle (*Theo.* 38.) ought to consist of less than 4 right Angles, and 3 plain Angles are the fewest that can make up a solid one (as is naturally evident) Now three Angles of a *Hexagon* are equal to four right, and in all Figures of more sides than a *Hexagon* they exceed four right. Wherefore 3, 4 and 5 Triangles (for 6 Angles of an Equilateral Triangles make 4 right) 3 Squares, and 3 Pentagons (which make $\frac{2}{3}$ of 4 right Angles) may all compose a solid Angle; And therefore there can be but 5 Regular Bodies, viz. A *Tetraedron*, whose Angle is made up of 3 Equilateral Triangles; A *Cube*, the Angle of which is formed by 3 Squares; An *Octaedron*, whose Angle is made up of 4 Equilateral Triangles; A *Dodecaedron*, whose Angle is formed by 3 Pentagons; And and an *Icosaedron*, whose Angle consists of 5 Equilateral Triangles.

If you cut out the following Figures in Pastboard and fold them up, they will represent the aforesaid 5 Regular Bodies.

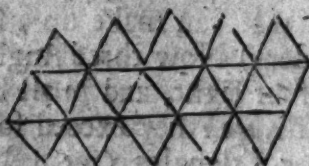
The Five Regular Bodies.



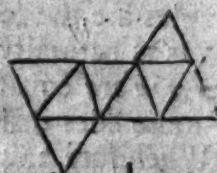
cube



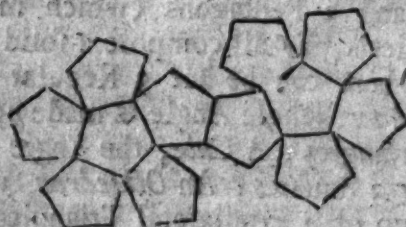
tetraedron



icosahedron



octahedron



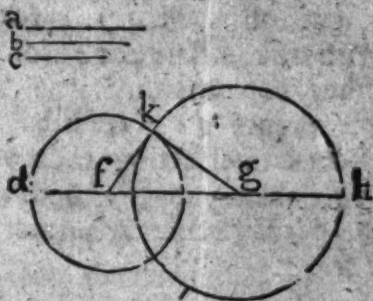
Dodecaedron

Geometrical Problems.

PROBLEM I.

HOW of three straight lines abc to make a Triangle. Of which it is necessary that any two of them taken together be longer than the third.

On the infinite line dh take (post. 3.) df, fg, gb equal to the given lines c, a, b , and from the Centers f, g , with the Extents fd, gb describe two Circles cutting each other in k ; draw fk, kg and the Triangle $fk g$ shall be made, whose sides (post. 3. def. 6.) are equal to the 3 right lines given, *W. W. to be done.*



Schol. In like manner may be described either an Equilateral or Iſosceles Triangle, if so be that for the first, the 3 Circles be equal, and for the later two of them.

Use. Upon this Prob. depends the method, not only of making one Figure equal to another, but also of enlarging or diminishing the same. For having divided the Figure into Triangles, measure each side, and set down each sides measure upon it self, then according as you would have the Figure, either equal, greater or lesser, take from a scale (that is equal greater or lesser) the number

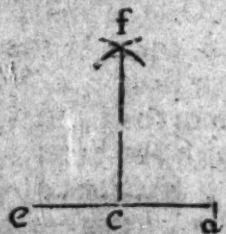
of parts that each of these sides contain, and with these Extents make Triangles, as in the preceding Problem is taught.

So that upon this Proposition, the business of Enlarging or Diminishing any Ground plot or Draught depends; that is, of redrawing it from a greater to a lesser, or from a lesser to a greater Plain.

P R O B. II.

From any given Right line as ed , and point c therein assign'd, to raise a Perpendicular.

On each side the given point c , cut off (*post. 3.*) $cd = ce$ with the distance de (or any other not less than dc) from the points d, e , describe two Arches intersecting each other in f , then join fc , and it is the perpendicular required. The *Dem.* depends on *Theo. 2.*



P R O B. III.

If the Perpendicular had been required to be raised in or near the end of the Line de , suppose the point e .

With any small distance set one foot of the Compass in e , and extend the other (any where on that side the line de , the Perpendicular is to be raised as) to b , on b as a Center describe with the space be the Semicircle $ae f$, drawing from a (the point where the Arch cuts the line de) thro' the Center b , the Diameter abf , then join the points fe , it gives the Perpendicular required. The *Dem.* depends upon *Cor. 2. Theo. 12.* where 'tis



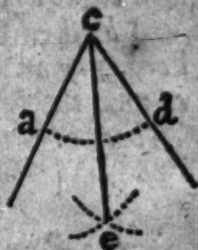
'tis prov'd that an Angle in a Semicircle is right.

Use. 'These Problems are of great use with most 'Mechanick Artificers; The drawing of Perpendiculars being very frequently required.

P R O B. IV.

To divide a Right lin'd Angle c, into two equal parts.

Set one foot of the Compasses in *c*, and strike the Arch *a d*, this cuts from each leg an equal part, on the points *a* and *d* with the same or any other distance, describe the prick'd Arches intersecting each other in *e*, then join *ce* and it performs what was required: For the Angle $a c e = e c d$. The *Dem.* depends upon *Theo.* 2.



Cor. Hence it appears how an Angle may be cut into 4. 8. 16 &c. equal parts, viz. by a continual bisection of each part. But the manner of cutting Angles into any number of equal parts, is as yet unknown to *Geometricians*.

Use. 'This Problem is very useful in dividing the 'Limbs of several Instruments, as *Theodelites*, *Semicircles*, *Quadrants*, and the like, since 'tis the same thing to divide an Arch as an Angle.

P R O B. V.

To cut a Right line a d into two equal parts.

On the ends *a* and *d* of the given right line, describe with any distance two Arches as *c a f*, *c d f*, join the intersections *c f*, with the right line *cf*, and it shall divide the given line *a d* equally in *e*. The *Dem.* depends upon *Theo.* 2.



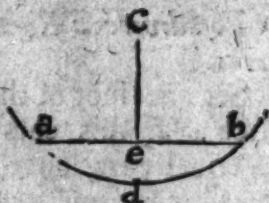
Use.

Use. 'This Prob. is very necessary for the dividing of Measures, and making of Scales.

PROB. VI.

From a point above as *c*, to let fall a Perpendicular to a given line *a b*.

From *c* with any distance greater than *c e*, describe an Arch of a Circle, cutting the given right line in the points *a* and *b*, then bisect *ab* in *e*, (Prob. 5.) and draw the line *c e*, and the thing is done. The Dem. depends up-

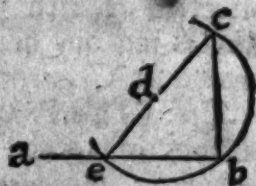


on Theo. 2.

PROB. VII.

From a Point *c* that is over the end of a given line as *a b*, to let fall a Perpendicular.

From *c* to any point as *e* in the line *a b* draw the straight line *c e*, which bisect in *d*, on *d* as a Center with the distance *d c* or *d e* describe the Semicircle *e b c*, and join *c b*, so is the right line *c b* the Perpendicular required. 'Tis Dem. after the same manner as was Prob. 3.



Use. 'These Problems are of great and general use by all sorts of Artificers; Also in Surveying, Fortification, Dyalling, and several other parts of the Mathematicks.

PROB. VIII.

To draw a right line *a b*, parallel to a given right line *c d*, and through *b*, a point assigned.

This

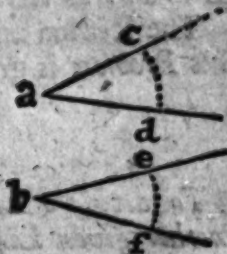
This case is universal, because all others fall un-
it, and the practice and Demonstration is shown in
the use of (*Theo. 6.*)

Use. ' Parallel lines are of great use in most parts
' of the Mathematicks, particularly in *Perspective*,
' *Navigation*, *Surveying* and *Dyalling*.

PROB. IX.

To make a right lin'd Angle b , equal to a right
lin'd Angle given a .

Draw the Right Line bf , set one foot of the
Compasses in the point a , and at
any convenient distance describe
the Arch cd , also with the same
extent set one foot in b , and de-
scribe the Arch ef . This done
take between your Compasses
the distance cd , and set from f
to e in the other Arch, and draw
 be it forms the Angle required,
the Demonstration of which, depends upon (*Theo. 2.*)



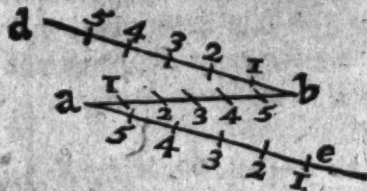
' This Prob. is so absolutely necessary that with-
' out the Knowledge of the practice hereof, nothing
' could be perform'd either in *Surveying*, *Fortifica-*
' *tion*, *Perspective*, *Dyalling*, *Drawing*, &c.

PROB. X.

To divide a Line $a b$ into any number of equal
Parts.

Suppose it be required to divide the Line ab in-
to 6 equal parts, draw from b the Line bd , making
an Angle at pleasure with the Line ab , also from a
the contrary way and side, draw the Line ae , making
the same with Angle ab , as the former Line bd did;
then run any small distance 5 times from b on the
line

line bd , which must always be one less than the number you intend to divide the Line into, do the like with the same space on the Line ae , but the contrary way. Then lay a Ruler from the Figure 5 in each Line, it shall cut the Line ab in 1.



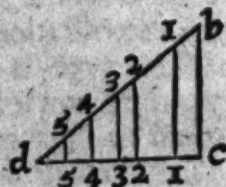
Proceed thus, by laying the Ruler over 4, 4. 3, 3, &c. and it will divide the Line ab into 6 equal parts as was required.

The Reason and Demonstration of the practice of this Problem is grounded upon the 23d Theo.

PROB. XI.

To divide a Line db after the same manner as another given Line dc is divided.

Make with the two Lines db , dc the Angle bdc at pleasure, Join the points bc and from the Points 5. 4. 3. 2. 1. in the Line dc draw Lines 1, 1. 2, 2. 3, 3. &c. parallel to bc , they will divide db after the same manner as dc was before divided. The Dem. of this does also depend upon the 23d Theo.

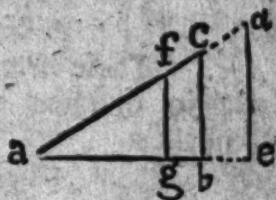


Use. ' Both this and the former Prob. is necessary for the division of Instruments.

PROB. XII.

Two Right Lines ab , ac being given to find a third proportional, either encreasing or diminishing.
Make

Make with the two given Lines an Angle at pleasure, as bac , join bc , then (if you would have the proportion encreasing) produce ab , so that $ae = ac$, from e parallel to bc draw ed producing ac till it meet in the point d , so is ad the fourth proportional encreasing; for as $ab : ac :: ae (ac) : ad$ Theo. 23.

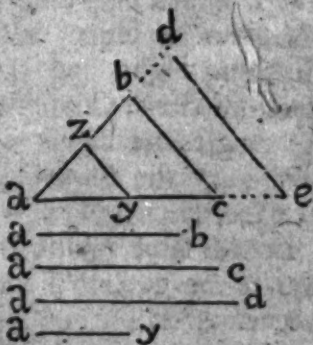


If you would have it diminishing, then make $af = ab$, and from f draw fg parallel to cb , so is ag the fourth proportional diminishing; for as $ac : ab :: af (ab) : ag$ Theo. 23.

P R O B. XIII.

Three Right Lines ab , ac , ad being given, to find a fourth proportional, either Encreasing or Diminishing.

If you would find it Encreasing, make an Angle at pleasure with the two shortest Lines ab , ac , produce ab till $abd = ad$, the longest of the three given Lines; from d draw de parallel to bc , producing ac till it meet in e , so is ae the fourth proportional: For as $ab : ac :: ad : ae$ Theo. 23.



If you would find it diminishing, let ab , ac be the two longest of the three given Lines, and ay the shortest, then having formed an Angle with the two

two longest of them, cut from $a c$, $a y$ equal to the least of three given Lines, from y draw $y z$ parallel to $b c$, so is $a z$ the fourth proportional Diminishing; for as $a c : a b :: a y : a z$. *Theo. 23.*

Use. These Problems are very useful, particularly in dividing the Sector and proportional Compasses.

P R O B. XIV.

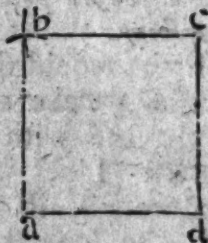
Two Right Lines being given, to find a mean Proportional.

See *Schol.* of *Theo. 29.*

P R O B. XV.

Upon a given Right Line $a d$, to make a Geometrical Square.

Erect from d the Perpendicular $d c = a d$, on the points c, a describe two Arches intersecting one another in b , join $a b, c b$, and the thing is done. The *Dem.* is clear, for one Angle being right, and the 4 sides equal by *Construction*, the rest of the Angles are right; wherefore (*def. 24.*) the Figure $a b c$ is a true Geometrick Square.



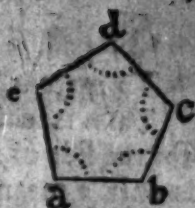
By the same method may the Rectangle or long Square be describ'd, if the distance $d c$ be taken greater or lesser than $a d$.

P R O B. XVI.

Upon a given Line $a b$, to make a Regular or Equilateral Pentagon $a b c d e$.

Find the Degrees contained in one Angle, which
(*Theo 4.*

(*Theo.* 4. *Schol.* 2.) are 108 deg. on the point b , make (*Prob.* 9.) an Angle of an 108 deg. with the given line $a b$, and take $b c = a b$, on the point c make another Angle of 108 deg. and make $c d = b c$; proceed thus with the rest of the Angles and sides till you come to the Angle e , where you need but join the points $e a$ and the Figure is compleated, For the Angles e and a are easily prov'd equal to the rest; Wherefore (*def.* 31.) the Figure $a b c d e$ is a regular *Pentagon*.



In like manner may any other regular *Polygon* be described upon a given Line, by laying down the Angle as found (*Theo.* 4. *Schol.* 2.) and so enclosing the Figure with equal sides.

P R O B. XVII.

To find a Center whereby to strike a Circle that shall pass through any three Points, as $b c d$, provided they are not placed in a strait Line.

Join the three points together by the strait Lines $d b$, $d c$, $b c$, cut any two of those three Lines, suppose $d c$ and $b c$, into two equal parts. From the points of cutting and Perpendicular thereto, draw the right Lines $e a$, $g a$; the point a where these Lines cut each other is the Center of the Circle; From which if with the distance $a b$, $a c$ or $a d$ you describe a Circle, it shall pass through the three given points $b. c. d$. as was required. The *Dem.* depends on *Theo.* 10



P R O B.

P R O B. XVIII.

To make a Square equal to a given Rectangle.

This is easily deducible from the 29th Theo. for the side of a Square equal to any given Rectangle, is only a mean proportional betwixt the longest and shortest side of it. See also the 14th Prop. of Euclid's Second Book.

P R O B. XIX.

To make a Square equal to the Sum or difference of any two Squares given.

See the Practice and Demonstration of it, in the use of Theo. 9.

P R O B. XX.

To divide a Parallelogram into two equal parts, by a Line that shall pass through any assigned point within the Figure.

See the Practice and Dem. of this Problem in the use of Theo. 6.

P R O B. XXI.

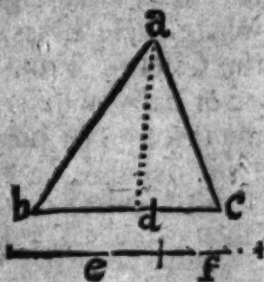
To divide a Triangle into any number of equal Parts.

See the Practice and Dem. of this in the use of Theo. 8.

P R O B. XXII.

To divide a Triangle a b c into two parts by a Line drawn from an Angle a, in such proportion as the Line e to the Line f.

Divide the side b c (Prob. 11.) in such proportion as e to f. From a draw the Line a d and it is done; for as $e : f :: b d : d c$ (Con.) : $\triangle b a d : \triangle d a c$. Theo. 22.



Use

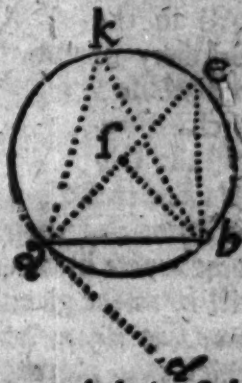
Use. 'The three preceding Propositions are of great use, particularly in *Surveying*, where 'tis often requir'd to cut from any Field, or measure any parcel of Arches.

P R O B. XXIII.

Upon a given Right Line ab to describe the Segment of a Circle $akeb$, which shall contain any given Angle, suppose 40 deg .

Through the point a draw the right Line ad , making an Angle of 40 deg . with the Line ab : From a draw ae , perpendicular to ad , and join eb ; At the point b make the Angle $abf = baf$; from the point f as a Center with the distance fa , describe a Circle which shall also pass through b , because the Angle $fab = fba$, then I say that the Segment $ae b$ is the Segment sought. For draw from the points a, b , two Right Lines bk, ak to form an Angle in any part of the Periphery as at k , the Angle $akb = bad$, which is equal to 40 deg . by Construction.

Dem. The Line ae is a Diameter of the Circle by (Theo. 14.) and therefore the Angle $e b a$ is right, and consequently $\angle e + e a b = \text{one right}$. But $\angle e a b + b a d = \text{one right}$ (Theo. 1.) therefore $\angle e = b a d$, but $\angle e = b k a$ because they stand on the same Arch; therefore $akeb$ is the Segment requir'd



Use. 'Upon this Problem is grounded the Solution of a very useful Proposition, incerted in the *Philosophical Transactions*, Num. 69. page 2087, and that after a more easie and universal Method than is there express'd. It is as follows.

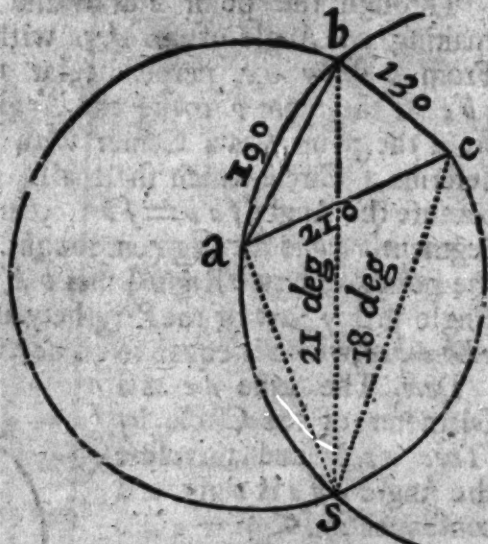
G

The

The distance between any 3 Objects being known and the Angle they make with each other from some known Point or Station in the same Plain, to find the distance of either of those Objects from the said Station.

Let the three Objects be a , b , c , and their Distances and Angles as in the annexed Diagram; upon the Line ab describe by the preceding Prob. the Segment (asb) of a Circle that shall contain an Angle of 21

deg. also upon the Right Line bc describe the Segment of a Circle that shall contain an Angle of 18 deg. Now since I must stand in the Periphery of both these Circles to behold the Ob-



jects under those Angles, 'tis evident it must be in the Point s of their Intersection: For if from the point s you draw the Right Lines sa , sb , sc , they will form the given Angles, as is manifest from the preceding Prob. And therefore if by the Scale, with which you plotted the Triangle abc , you measure the Lines sa , sb , sc , you will get the distance of the said point from either of the Objects, which was the thing required.

Use. 'By the help of this Prob. we may likewise find the true position of Sands and Rocks that are near the Shoar, or within sight of three places at Land whose Distances are known.

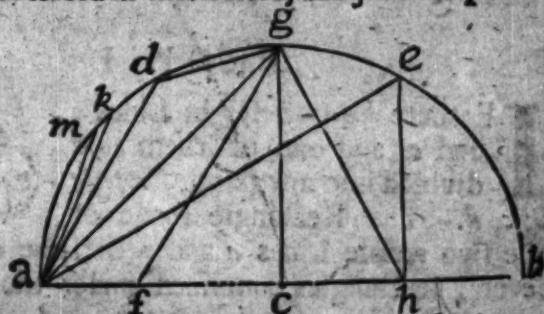
P R O P.

P R O B. XXIV.

To divide a Circle into 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 equal parts, that is to describe an Equilateral Triangle, Square, Pentagon, Hexagon, &c. in a Circle.

Describe a Circle and draw through the Center the Diameter $a c b$, then take the distance $c a$ and set on the Circumference from a to d , and from thence again to e , and draw $a d$, $a e$ the first for the side of an Equilateral Triangle, the second for the side of a Hexagon; cross $a b$ at right Angles with the Line $c g$, and draw $a g$ for the side of the Square; bise $\bar{c}$ t $c b$ in h , and from g draw $g h$, make $b f = b g$, and draw $f g$ for the side of a Pentagon inscribed in this Circle, and join the points $b e$ it gives

the side of a Heptagon; next bise $\bar{c}$ t the Arch $a g$ in k , and draw $a k$ for the side



of the Octagon, this done divide the Arch $a d e$ into 3 equal parts at m , and draw $a m$ for the side of an Enneagon or 9 sided Figure; the Line $f c$ is the side of a Decagon, for the side of Endecagon or 11 sided Figure; you must have recourse to Mechanick Artifices, that is, to divide the whole Circle into 11 equal parts, one of which will be the side required. Lastly, join the points $d e$ it gives the sides of the Dodecagon or 12 sided Figure.

Many other Probs. there are in Practical Geometry, which might sometimes be required, but I omit them, having only incerted those that are most frequent and useful, and generally required in the Delineating of any Geometrick Scheme or Figure.

EUCLID's

Second BOOK,

Geometrically Constructed and Algebraically Demonstrated.

PROP. I.

IF there be two Right Lines a and e , and one of them as a divided into any parts, *suppose* b, c, d . The Rectangle made of the two whole Lines a and e shall be equal to the several Rectangles contained under the whole Line e , and the Segments or parts b, c, d .



That is, $ae = be + ce + de$
Solution Algebraically. Let $a = b + c + d$
 Multiply each by e
 it produces $ae = be + ce + de$

Schol. Hence if there be two right lines a, e , and both of them cut into several parts *suppose* a into the parts b, c, d and e into f, g . The Rectangle under the two whole right Lines, shall be equal to the several Rectangles under all the parts.



That

(89)

That is, $ae = fb + fc + fd + gb + gc + gd.$

Sol. Alg. Let $a = b + c + d$

and $e = f + g$ by which *Mult.*

it produces $ae = fb + fc + fd + gb + gc + gd.$

PROP. II.

If a Right Line a be cut any how into two parts as bc , the Rectangles comprehended under the whole Line a , and each Segment b and c are equal to the Square made of the whole Line a .



That is, $aa = ab + ac$

Sol. Alg. Let $a = b + c$

Mult. each by — — — a

it produces $aa = ba + ca$

PROP. III.

If a right Line a be any way divided into two parts as bc . The Rectangle comprehended under the whole Line a , and one Segment as c , shall be equal to the square of the said Segment, and a Rectangle of both Segments.



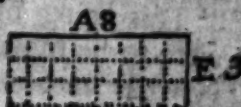
That is, $ac = bc + cc$

Sol. Alg. Let $a = b + c$

Mult. each by — — — c

it produces $ac = bc + cc$

Use. 'This with the two foregoing Propositions serve to demonstrate the ordinary or common Rule of *Multiplication*; Thus, Let
' A be one given Line and the
'Line noted by E another. If



G 3

then,

then, (as in *Theo.* 7. has been hinted) the Line *E* be placed perpendicular to the Line *A*, and in that posture be moved parallel to it self, through every *Physical Point* in the Line *A*, it will produce the the Rectangle *AE*; and that this motion representeth Multiplication, doth thus appear.

Let there be assumed in the Line *A* any number of points Mathematical, suppose 8, and in the Line *E* let there be taken 3; Now 'tis evident that in the Rectangle *AE*, there shall be as many Lines equal to *E* as there are points in the Line *A*: Multiplying therefore 8 by 3 (*that is* the number of points in the Line *A*, by those in the Line *E*) it will produce 24 for the number of *Mathematick Points* in the whole *Parallelogram AE*; And this is the reason of that common *Notion* in Multiplication, which is that the *Multiplicand* must be put so often into it self as there are Units in the *Multipplier*, for the Line *A* is increased in breadth so often as there were points taken in the Line *E*.

But here note, That by a Mathematical point must be understood some part or quantity of a Line, and not such a point as hath no part. For Example, If I suppose the Line *A* to contain 8 yards, 8 feet, or 8 inches, then 1 yard, 1 foot or 1 inch I term a Point, without considering that the same is composed of parts; For in measuring any Line, I must make use of some Line whose length is known, so likewise in the measuring of Superficies and Solids, use must be made of a Superficies and Solid whose Dimensions are known.

This being premised, suppose the number 43 were to be multiplied by the number 6, having divided 43 into two parts, *viz.* 40 and 3, I multiply 40 by 6 it produceth 240, also I multiply 3
by

'by 6 it produceth 18, which two Rectangles 240
'and 18 are equal to the whole Rectangle of 43 by
'6, that is to 258: The reason of which equality
'is grounded on this *Axiom*, That if equal things be
'multiplied by the same or equal things, the products
'shall be equal.

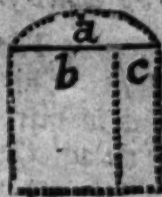
$$\text{For } 43 = 40 + 3$$

$$\text{Mult. each by } \underline{\quad 6 \quad}$$

$$\text{it produces } \underline{258 = 240 + 18}$$

P R O P. IV.

If a right line a be any how cut
into parts, as b and c , then shall the
square of the whole line a be equal
to both the squares of the Segments
 b and c , and to a double Rectangle
of the said Segments b and c .



$$\text{That is, } aa = bb + 2bc + cc$$

$$\text{Sol. Alg. Let } a = b + c$$

$$\text{Mult. by } \underline{\quad a = b + c \quad}$$

$$\text{it produces } \underline{aa = bb + 2bc + cc}$$

Cor. Hence it follows that the square of half any
line is equal to the $\frac{1}{4}$ of the square of the whole line;
For the square of $\frac{1}{2} a$ is $\frac{1}{4} aa$.

Use. 'Upon this Proposition depends the method
'of Operation in the Extraction of the square Root.

P R O P. V.

If a straight line a be cut into two equal parts
each $\frac{1}{2} a$, and into unequal parts,
viz. $\frac{1}{2} a + b$ and $\frac{1}{2} a - b$, then is the
Rectangle of the unequal parts
with the square of the difference of
the equal and unequal parts, equal to the square
of half the line a .



G 4

That

That is $\frac{aa}{4} = \frac{a}{2} + b \times \frac{a}{2} - b + bb$

Sol. Alg. Let $\frac{a+b}{2}$ be the greater part

Then $\frac{a-b}{2}$ is the lesser part.

the Product of which is $\frac{aa}{4} + \frac{ab}{2} - \frac{ab}{2} - bb$

add to this bb it makes $\frac{aa}{4} + \frac{ab}{2} - \frac{ab}{2} - bb$

$+ bb$ which is evidently equal but to $\frac{aa}{4}$ when the destroying Terms are thrown away.

Use. This with the preceding Proposition is used in demonstrating the method of finding the Root of an affected Equation.

PROP. VI.

If a straight line a be cut into two equal parts each $\frac{a}{2}$ and another right line b be put to the same directly in one strait line, then is the Rectangle of the whole and the line added taken as one line, and the line added, together with the Square of $\frac{a}{2}$ equal to the square of $\frac{a}{2} + b$ taken as one line.



That is $a + b \times b + \frac{aa}{4} = \frac{a}{2} + b \times \frac{a}{2} + b$

Sol. Alg. $a + b$

Mult. by b

it produces $ab + bb$ add to this $\frac{aa}{4}$

it makes $ab + bb + \frac{aa}{4}$ which is equal to

the square of $\frac{a}{2} + b$

For

For $\frac{a}{2} + b$

Mult. by $\frac{a}{2} + b$

makes $\frac{aa}{4} + ab + bb$

PROP. VII.

If a straight line a be any wise cut into two parts, as b and $a-b$, then the square of the whole line added to the square of one of the parts, is equal to two Rectangles under the whole line a , and the said part b , with the square of the other part or Segment $a-b$.



That is, $aa + bb = 2ab + a-b \times a-b$.

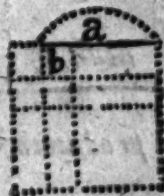
Sol. Alg. $a-b$

Mult. by $a-b$

it produces $aa - 2ab + bb$ add to this $2ab$
it makes $aa - 2ab + bb + 2ab = aa + bb$.

PROP. VIII.

If a straight line a be cut any how into two parts as b and $a-b$, then four Rectangles of the whole line and one of these parts with the square of the other part, shall be equal to the square of the whole line and first Segment taken as one line.



That is, $4ab + a-b \times a-b = a + b \times a + b$

Sol. Alg. $a-b$

Mult. by $a-b$

it produces $aa - 2ab + bb$ add to this $4ab$
it makes $aa - 2ab + bb + 4ab$ which is
equal to the square of $a+b$.

For

For $a + b$
 Mult. by $a + b$
 makes $a a + 2 a b + b b$ equal to the former.

P R O P. IX.

If a straight line a be cut into two equal parts, each half a , and into unequal parts as b and $a - b$, then are the squares of the unequal parts b and $a - b$ double to the Sum of the square of half a , and the square of $\frac{a}{2} - b$ the difference.



That is, $b b + \frac{a - b \times a}{2} = \frac{a a}{4} + \frac{a}{2} \times \frac{a}{2} - b \times \frac{a}{2} + b b$.

Sol. Alg. $a - b$

Mult. by $a - b$

it produceth $a a - 2 a b + b b$ add to this $b b$.
 it makes $a a - 2 a b + 2 b b$

Again $\frac{a}{2} - b$

Mult. by $\frac{a}{2} - b$

it produceth $\frac{a a}{4} - \frac{a b}{2} + \frac{a b}{2} + b b$ add to this $\frac{a a}{4}$

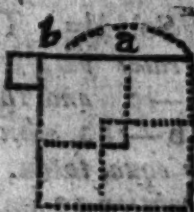
it makes $\frac{a a}{2} + b b$ which is but half the former.

Use. This with the foregoing Propositions are principally used in *Algebra*, for finding the Root of a Square equal to a Number encreased more by a certain number of Roots.

P R O P.

PROP. X.

If a straight line a be cut into two equal parts, each half a , and another right line b be added to the same directly in one strait line, then is the square of the whole line and the line added taken as one line, double to the Sum of the squares of half a , and $\frac{1}{2} a + b$ taken as one line.



the

$$\text{That is, } \frac{a+bx}{2} + b = \frac{1}{2}a + bx + \frac{1}{2}a + b + \frac{1}{2}aa$$

Sol. Alg. $a+b$

Mult. by $a+b$

it produces $a^2 + 2ab + bb$ add to this b^2
it makes $a^2 + 2ab + 2bb$.

Again $\frac{a}{2} + b$

Mult. by $\frac{a}{2} + b$

it produces $\frac{a^2}{4} + \frac{ab}{2} + \frac{ab}{2} + bb$ add to this $\frac{a^2}{4}$

it makes $\frac{a^2}{2} + ab + bb$ which is but half the former

PROP. XI.

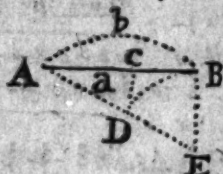
To cut a line as b into *extream* and *mean* Proportion, *that is*, to cut a line into two such parts, that the square of the greater shall be equal to a Rectangle made of the whole line, and lesser parts.

That

That is $a^2 = b^2 - a \times b$.

Sol. Alg. Take the thing for done, and let the greater part be noted by a , then will the lesser be $b - a$, and the Product of the greater and lesser $bb - ba$ which according to the Proposition must be equal to aa , that is $bb - ba = aa$, but by Transposition $aa + ba = bb$, then by adding to either part of the Equation the square of half b the co-efficient, to compleat the square it will make $aa + ba + \frac{1}{4}bb = bb + \frac{1}{4}bb$, and then extracting the square Root of each, we have $a + \frac{1}{2}b = \sqrt{bb + \frac{1}{4}bb}$ or $a = \sqrt{bb + \frac{1}{4}bb} - \frac{1}{2}b$.

Geometrically, Let the given line be $AB = b$, from B erect $BE = \frac{1}{2}AB$ perpendicular to it, and and let be drawn AE ; Then (Theo. 9.) is $AE = \sqrt{bb + \frac{1}{4}bb}$; on the Center E with the distance $EB = \frac{1}{2}b$ describe the Arch BD this cuts from the line AE the part $ED = \frac{1}{2}b$, and then the remaining part AD is equal to $\sqrt{bb + \frac{1}{4}bb} - \frac{1}{2}b = a$. And therefore $AC (= AD)$ is the part or Segment sought.

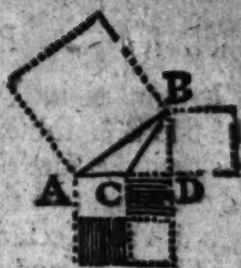


Use. This Proposition is requisite for the Inscription of some of the Regular Bodies in a Sphere. It is also necessary to inscribe a Pentagon in a Circle, as also a Pentecagon, and many other necessary Practices it helpeth to perform. Father Lucas of St. Sepulchres composed a whole Book of a line cut in this Manner.

PROP. XII.

In an obtuse angled Triangle ACB , the square of

of the side AB subtending the obtuse Angle, is greater than the sum of the squares of the sides CA , CB , including the said Angle, by a double Rectangle made of the side CA , and that part CD betwixt the said obtuse Angle and the point D , where the perpendicular BD falleth.



That is, $AB^2 = AC^2 + CB^2 + 2ACD$.

Sol. Alg. By *Theo. 9.* $AB^2 = AC^2 + 2ACD + CD^2$, and by the same $CB^2 = CD^2 + BD^2$, therefore (*ax. 2.*) $AB^2 = BD^2 + CB^2 + 2ACD$. *W. W. D.*

Use. 'By this Proposition the Perpendicular, and consequently the Area of any Triangle may be found by having its three sides.

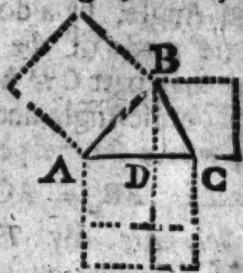
'As suppose AB be 40 foot, AC 26 foot, and BC 22 foot; then I say the square of AB will be 1600, the square of AC 676, BC 484, the Sum of the two latter squares 1160, which being taken from 1600 Leaves 440 for the two Rectangles under AC , CD , the one half of which is 220, for one of these Rectangles, which divided by AC 26, will give $8\frac{4}{13}$ for the Line CD , whose Square is $71\frac{16}{169}$, which being subtracted from the square of BC 484, there remains the square of BD 412 $\frac{53}{169}$, the Root of which is 20.3 *prope* for the Perpendicular BD .

'Having thus obtain'd the Perpendicular, 'tis easie to find the Area of the Triangle, 20.3 the Perpendicular; For multiplied by 13 which is half the Base, will give 134.9 the superficial Content of the Triangle.

P R O P.

PROP. XIII.

In any plain Triangles as ABC the square of the side BC subtending any acute Angle, as A , is less than the Sum of the squares of the other two sides AB, AC by a double Rectangle made of the side CA , and the Segment DA lying betwixt the said acute Angle and the point D where the Perpendicular falleth.



That is, $BCq = BAq + CAq - 2 CAD$,

Sol. Alg. By *Theo.* 9. $BAq - DAq = BCq (-D Cq) - CAq + 2 CAD - ADq$ (*Prop.* 7.) as is very evident by changing the Signs of $-DC$ and its equal $-CAq + 2CAD + ADq$ into their contraries. And therefore $BAq = BCq - CAq + 2 CAD$. So that (*ax.* 2.) $BAq + CAq = BCq + 2 CAD$ or $BCq = BAq + CAq - 2 CAD$. *W.W.D.*

Observe that the Letters are so order'd as to agree with the Figure of *Prop.* 12. in case there be an obtuse Angle in the Triangle.

Note also in both cases, that the double Rectangle of $DA C$ added to the square of BC , or its equal $BDq + DCq$ make the two squares of the Lines AB, AC .

Use. This, as also the preceding *Prop.* are used in the solution of several *Algebraick* Questions; they are likewise very necessary in *Trigonometry*.

PROP. XIV.

To find the side of a square, equal to a given Rectangle $ABCD$.

Gen

Geo. To F produce A B, making $BF = BD$, and bisect A F in E, on E with the Radius A E or E F describe the Semicircle A G F, produce D F till it cut the Circle in G, then (*Theo.* 29.) is G B the side of a Square equal to the given Rectangle.

Sol. Alg. Draw E G and let $AF = b$ and $EB = a$, then is $AE = EF = GF = \frac{1}{2}b$, Also $AB = \frac{1}{2}b + a$, and $BD = BF = \frac{1}{2}b - a$, and the Rectangle A D, that is A B D equal to $\frac{1}{2}b + a \times \frac{1}{2}b - a$, that is $\frac{1}{4}bb - aa$ (*Theo.* 9.) For the square Root of $\frac{1}{4}bb$ is $\frac{1}{2}b = EG$, and the square Root of aa is $a = EB$, therefore G B is the side of a square equal to the Rectangle A D. W.W.D.



The Doctrine of Proportion, in its various Changes and Combinations, Algebraically Demonstrated.

IF Four Magnitudes are proportional, they shall also be proportional in *Alternation, Inversion, Composition, Division, Conversion and Mixtion.*

For if $A : a :: B : b$.

Then by	{	Alternation $A : B :: a : b$
		Inversion $a : A :: b : B$
		Composition $A + a : a :: B + b : b$ or $A + B : B :: a + b : b$
		Division $A - a : a :: B - b : b$ or $A - B : B :: a - b : b$
		Conversion $A : A + a :: B : B + b$
		Mixtion $A + a : A - a :: B + b : B - b$, or as $A + B : A - B :: a + b : a - b$.

PROP. I.

If 4 Quantities are proportional, the Product or Rectangle of the first and fourth, is equal to the Product of the second and third.

Suppose $a : b :: c : d$, then I say that $a \times d = b \times c$, that is $ad = bc$.

Dem. For (def. 48.) $\frac{a}{b} = \frac{c}{d}$ multiply both by b , it will make $a = \frac{bc}{d}$ and again multiply both by d , it will make $ad = bc$. W.W.D.

For Equal things, multiplied by the same or equal things, gives equal Products, upon which Ax. the Demonstration both of this, and the following Propositions about Proportion usually depend.

In like manner if there be 3 Quantities in continual proportion, the square of the mean is prov'd equal to the Product of the Extreams.

This Proposition shows the reason of the operation in the Common Rule of Three, where we multiply the second Term by the third, or third by the second, and divide the Product by the first to get the fourth; For if the second multiplied by the third, be equal to the first multiplied by the fourth, as is here prov'd, then 'tis manifest that the Product of the second and third divided by the first shall give the fourth proportional, for it gives a Number, which if the Divisor multiplies gives the Dividend.

PROP. II.

Numbers or Quantities that have the same Proportion to a third, have also the same one to another.

Suppose $a : b :: c : d :: e : f$, then I say $a : b :: e : f$.

Dem.

Dem. For $\frac{a}{b} = \frac{e}{d} = \frac{f}{g}$ (def. 48.) and consequently $\frac{a}{b} = \frac{f}{g}$ and therefore $a : b :: e : f$. W.W.D.

P R O P. III.

If four Quantities are proportional, they shall be so in Alternation.

Suppose $A : a :: B : b$, then I say it will hold Alternately, as $A : B :: a : b$.

Dem. For $Ab = aB$ (Prop. I.) divide both by bB the Quote is $\frac{Ab}{bB} = \frac{aB}{bB}$ or $\frac{A}{B} = \frac{a}{b}$ and therefore $A : B :: a : b$. W.W.D.

P R O P. IV.

If four Quantities are proportional, they shall be so by Inversion.

Suppose $A : a :: B : b$ then inverfly it will hold as $a : A :: b : B$.

Dem. For $Ab = aB$, divide each by AB the Quote is $\frac{Ab}{AB} = \frac{aB}{AB}$ or $\frac{b}{B} = \frac{a}{A}$ and therefore as $a : A :: b : B$. W.W.D.

P R O P. V.

If four Quantities are proportional, they shall also be so by Composition and Division.

Suppose $A : a :: B : b$, then by Composition $A + a : a :: B + b : b$, and by Division $A - a : a :: B - b : b$.

Dem. For $\frac{A}{a} = \frac{B}{b}$ and then $\frac{A+a}{a} = \frac{B+b}{b}$ that is (reducing those mixt Quantities into Fractions) $\frac{A+a}{a} = \frac{B+b}{b}$ therefore $A \pm a : a :: B \pm b : b$, which

H

proves

Dem.

proves both *Composition* and *Division*, the *Alternations* and *Inversions* of which follow from what was before proved.

PROP. VI.

If four Quantities are proportional, they shall be so by Conversion.

Suppose $A : a :: B : b$, then by converse proportion it will hold as $A : A \pm a :: B : B \pm b$.

Dem. For $\frac{a}{A} = \frac{b}{B}$ (Prop. 4.) therefore $1 \pm \frac{a}{A} = 1 \pm \frac{b}{B}$

which by Reduction makes $\frac{A \pm a}{A} = \frac{B \pm b}{B}$ and therefore as $A \pm a : A :: B \pm b : B$, but invertly will hold as $A : A \pm a :: B : B \pm b$, which proves Conversion, the Alternation of which is prov'd (Prop. 3.)

PROP. VII.

If four Quantities are proportional, they shall also be so by Mixtion.

Suppose $A : a :: B : b$, then mixtly 'twill hold as $A + a : A - a :: B + b : B - b$.

Dem. For by *Conversion* $A : A \pm a :: B : B \pm b$, but by the *Inverse* of Division $a : A - a :: b : B - b$.

Now because $\frac{A}{A - a} = \frac{B}{B - b}$ and $\frac{a}{A - a} = \frac{b}{B - b}$ therefore

(ax. 1.) $\frac{A}{A - a} + \frac{a}{A - a} = \frac{B}{B - b} + \frac{b}{B - b}$ or $\frac{A + a}{A - a} = \frac{B + b}{B - b}$. And therefore $A + a : A - a :: B + b : B - b$. *W.W.D.*

After the like method may other Augmentations in Proportion, be prov'd by Alternating, Inverting and Combining of these.

PROP. VIII.

If like Proportionals be multiplied by like Proportionals, the Products shall be proportional.

Suppose $A : a :: B : b$, also $E : e :: F : f$, then I say 'twill hold as $AE : ae :: BF : bf$. *Dem.*

Dem. For $\frac{A}{a} = \frac{B}{b}$ also $\frac{E}{e} = \frac{F}{f}$ then $\frac{EA}{ae} = \frac{BF}{bf}$ and therefore as $EA : ae :: BF : bf$. W.W.D.

PROP. IX.

If there be two ranks of Quantities, suppose $A : B : C : D$ and $a : b : c : d$. Then if $A : B :: a : b$, also $B : C :: b : c$. Lastly, $C : D :: c : d$, then I say it will also hold as $A : D :: a : d$.

Dem. For because $\frac{A}{B} = \frac{a}{b}$ and $\frac{B}{C} = \frac{b}{c}$ also $\frac{C}{D} = \frac{c}{d}$ therefore $\frac{A}{B} \times \frac{B}{C} \times \frac{C}{D} = \frac{a}{b} \times \frac{b}{c} \times \frac{c}{d}$ For the Factors in each being equal, the Products are equal, Expung therefore like Terms in both Dividend and Divisor, and there will remain $\frac{A}{D} = \frac{a}{d}$ and therefore $A : D :: a : d$ which proves proportion of equality.

In like manner may Inordinate proportion be prov'd.

PROP. X.

If there be two ranks of Quantities, as in the Margin then I say $A : B :: C : D$ it will hold as $A : a :: d : D$.
 $a : B :: C : d$

Dem. For $AD = BC = ad$ (Prop. I.) therefore $AD = ad$; divide both by aD , the Quote is $\frac{AD}{aD} = \frac{ad}{aD}$ or $\frac{A}{a} = \frac{d}{D}$ And therefore as $A : a :: d : D$, which proves Reciprocal proportion.

PROP. XI.

If never so many Quantities are proportional, suppose $A : a :: B : b :: C : c$. Then as one Antecedent is to its Consequent, so is the sum of all the Antecedents to the sum of all the Consequents.

Dem. For if $A:a::B:b::C:c$, then $Ab = aB$ and $Ac = aC$: But $Ab + Ac = aB + aC$, that is, $b + c \times A = B + C \times a$; which turn'd into a proportion (*p. contra Prop. 1.*) will be as $A:a::B+C:b+c$, but by Alternation $A:B+C::a:b+c$, and by Conversion. $A:A+B+C::a:a+b+c$; and again by Alternation, as $A:q A+B+C::a:a+b+c$. *W.W.D.*

Hence therefore, if there be a rank of continual proportionals, suppose $A:b:c:d:e$, &c. to U , From the preceding Argumentation a *Theo.* may be easily rais'd, for finding the Sum of such a Progression.

As Let A be the first or least Term, U the last or greatest, and Z the sum of all the Terms, then $Z-U$ the sum of all the Antecedents, and $Z-A$ the sum of all the Consequents; and since by the preceding, One Antecedent is to its Consequent, as the sum of all the Antecedents, to the sum of all the Consequents; therefore as $A:b::Z-U:Z-A$, and therefore (*Prop. 1.*) $AZ-AZ = bZ-bU$, that is $bU-AZ = bZ-AZ$; and dividing both by $b-A$ the Quotes are $\frac{bU-AZ}{b-A} = Z$.

Whence arises this General Rule for finding the sum of a Geometrick Progression, divide the Product of the second and last, made less by the square of the first; by the second Term made less by the first, the Quote is the sum of the whole Progression.

Hence 'tis also evident, that if the first and second Terms are given, the *Ratio* is also given; As suppose A the first Term, and B the second, $\frac{A}{B}$ is the *Ratio*, that is, the Quantity of the *Ratio*. If only the first Term and Quantity of the *Ratio* is given, then the second Term is found by Multiplying the first Terms by the *Quantum* of the *Ratio*.

T H E

THE
Measuring of Superficies
 AND
SOLIDS:
Vulgarly, Decimally and Practically.

With the Customs used by Artificers, for Measuring their several Works. Likewise, Practical Directions for Measuring Board and Timber; Making of Vessels to hold any Quantity of Liquor, Corn, &c.

Of Measuring.

Meaſuring is the Art of finding the *Content* or *Quantum* of any ſort of Magnitude, from ſome given Dimensions.

Now before I give you the Rules for Meaſuring any ſort of Figure, its neceſſary to inform you that every *Magnitude* is meaſured by ſome *Magnitude* that is *Homogenial*, or like to it, or that is of the ſame kind with it; As a Line is meaſured by a Line, as one *Lineal, Inch, Foot, Yard, Rod, &c.* and a Solid is meaſured by a Solid, as one Solid or *Cubick, Inch, Foot, Yard, Rod, &c.*

So that when you know how many *Lineal Inches, Feet, &c.* are contained in a Line. Or *ſquare Inches, Feet, &c.* are contained in a Surface. Or *ſolid Inches,*

ches, Feet, &c. are contained in a Solid, then is the *Content* or *Quantum* of either of these *Magnitudes* said to be known. Other *Dimensions* besides these there are not in Nature, for *Length, Breadth* and *Depth*, include all *Space* or *Capacity*.

Hence 'tis evident, that every *Magnitude* must be measur'd by its *Homogenial* or like *Magnitude*; *that is*, *Longitude* by *lineal Inches, Feet, &c.* *Superficial Measure* by *square Inches, Feet, &c.* and *Solid Measure* by *solid Inches, Feet, &c.* Now the *square* and *solid Measure* is reduced from *Measures* of *Longitude*; *that is*, from a *Table of Long Measure*, may be made a *Table* both of *Superficial* and *Solid Measure*, after the following *Method*.

A Table of Long Measure.

<i>qu.</i>							
4	1.						
48	12	<i>Foot.</i>					
108	27	2 $\frac{1}{4}$	<i>El. F.</i>				
144	36	3	1 $\frac{1}{4}$	<i>Yard.</i>			
180	45	3 $\frac{1}{2}$	1 $\frac{3}{4}$	1 $\frac{1}{4}$	<i>El. En</i>		
782	198	16 $\frac{1}{2}$	7 $\frac{1}{2}$	5 $\frac{1}{5}$	4 $\frac{3}{4}$	<i>Rod</i>	

If you would know how many *square Quarters* there is in a *square Inch*, 'tis but multiplying 4 by 4; If you would know how many *square Quarters* there is in a *square Foot*, 'tis but multiplying 48 by 48.

Again, If you would know how many *solid Quarters* is in a *solid Foot*, 'tis but multiplying 4 by 4, and that product again by 4, it gives 64 the *solid Quarters* in one *solid Inch*. After this method may you *Calculate a Table of square and solid Measure*.

Now because I shall work most of the following *Examples*, both by *Vulgar* and *Decimal Arithmetick*, it will here be necessary to insert a *Table*, shewing what

what Decimal part of a Foot any number of Inches, Halves and Quarters are to 4 places, *that is to* $\frac{1}{16}$ part of a Foot.

The Decimal Table for Inches, Halves and Quarters.			
Inc.	Decim.	Inc.	Decim.
11	.9167	4	.3333
10	.8333	3	.25
9	.75	2	.1667
8	.6667	1	.0833
7	.5833	$\frac{1}{2}$	.0625
6	.5	$\frac{1}{4}$	.0417
5	.4167	$\frac{1}{8}$	.0208

By this Table you see that 9 Inches is .75 of a Foot, and 6 Inches is .5 of a Foot; If you would have the Decimal of 7 Inches $\frac{1}{4}$ or the like, then you must take the Decimal of 7 Inches, and also of $\frac{1}{4}$ of an Inch, and add them both together, so is the total the Decimal of 7 Inches $\frac{1}{4}$.

thus the De- $\left\{ \begin{array}{l} 7. \frac{1}{4} \\ 2. \frac{1}{2} \\ 3. \frac{1}{4} \end{array} \right\} \left\{ \begin{array}{l} .6041 \\ .2084 \\ .3125 \end{array} \right\}$
cimal of

But for such as would actually take their Dimensions Decimally, it will be necessary for those to provide themselves with a two Foot Rule, on which let each Foot be divided into 10 equal parts, and each of those parts into 10 other equal parts, so is the whole Foot divided into 100 equal parts, and is call'd by Artificers the Line of *Foot Measure*, being noted when put on the common two foot Rules, by the Letters F. M. This Rule so divided, is prepar'd to take any Dimensions in Feet and Decimal parts of a Foot; and therefore I shall suppose the following Dimensions to be taken by such a Rule that is in Feet and Decimal parts, as well as in Feet and Common Inches.

H 4

How

*How to find the Area of any plain Superficies,
both by Vulgar and Decimal Arithmetick.*

PROP. I.

To measure a true Square, long Square, Rhombus
or Rhomboids

Rule. Multiply their length by their true breadth,
the Product is the Superficial Content.

Ex. I. Suppose the side of a true Square be 17
Foot 6 Inches, what's the Content in square Feet.



Vulgarly.

$$\begin{array}{r}
 1. \\
 210 \text{ length.} \\
 210 \text{ breadth.} \\
 \hline
 2100 \\
 420 \\
 \hline
 44100 \text{ A. I.} \\
 \text{F. I.}
 \end{array}$$

$$144) 44100 (306.36 \text{ A.}$$

$$\underline{900}$$

$$\underline{36}$$

Decimally.

F. P.

$$17. 5.$$

$$17. 5.$$

$$\underline{875}$$

$$1225$$

$$\underline{175}$$

$$306.25 \text{ A. F. P.}$$

So

So that the Area or Content of this square, by the Vulgar way is 306 square Feet, and 36 square Inches; which 36, because $\frac{1}{4}$ of 144, is therefore $\frac{1}{4}$ of a Foot, exactly agreeing with the Decimal Area, that being 306 Foot, and 25 Inches, which is the $\frac{1}{4}$ of a Foot also.

Notes.

First, When Feet and Inches are given, they are reduced into Inches by Multiplying by 12, and taking in the odd Inches over and above the Feet; if you would reduce the Inches lower, *that is* into Halves, Quarters or half Quarters, Multiyly by 2, 4 or 8. This I thought necessary to premise, because, in Questions, I shall give the Dimensions not only in Feet and Inches, but also in Inches, Halves and Quarters, to save the trouble of reducing them, when the Question has such low Dimensions given in it.

Secondly, Having multiplied the Length and Breadth together in Inches; it gives the Content in square Inches; if the Length and Breadth were given in Halves or Quarters of Inches, then the Product will be square halves or Quarters of Inches: And so whatever Name the Dimensions are when you multiply them together, of that same Name is the Product, whether it be Feet, Yards, &c.

Thirdly, Because the Area or Content of Dimensions are generally required in Feet, Yards, or some higher Denomination, you must divide the square Quarters of Inches by 16, the Quote is square Inches: If in square half Inches you must divide by 4 to bring them to square Inches, and if you divide square Inches by 144, the Quote will give square Feet; and if you divide square Feet by 9, the Quote will give square Yards; understand the the like of reducing it to any other Denomination.

Fourthly,

Fourthly, Observe that A stands for Area, F for Feet, I for Inches, P for parts of a Foot, also Q for Quarters of Inches.

Ex. II. Suppose the length of a long Square be 25 Foot 9 Inches, or 309 Inches, and the breadth 13 Foot 3 Inches, or 159 Inches, what's the Content in square Inches.

159-I	309-I
-------	-------

Vulgarly.

309 length.
159 breadth.

2781

1545

309

49131 A. I.

F. I.

144) 49131 (341.27.A.

593

171

27

Decimally.

F. P.

25.75

13.25

12875

5150

7725

2575

341.1875 A. F. P.

Ex.

(111)

Ex. III. Suppose the side of a Rhombus be 8 Foot, 6 Inches, $\frac{1}{2}$ or 409 quarters of an Inch, and the true breadth or line A B 4 Foot, 3 Inches $\frac{1}{2}$ or 206 Quarters, what is the Area.



Vulgarly.

Q.

409

206

2454

8180

84254 A. in square Q.

16) 84254 (5265 A. in Sq. I.

42

105

94

14

F. I. Q.

144) 5265 (36. 81. 14. A.

945

81

Decimally.

F. P.

8. 52¹

4. 29¹

8521

76689

37042

34084

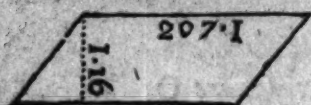
36.563611

A. F. P.

Here

Here (as also in some others of the following Example) observe that the Vulger and Decimal way do not exactly agree, the reason is because the 6 Inches and $\frac{1}{4}$, as also the 3 Inches and $\frac{1}{2}$ cannot be truly exprest in Decimals, and therefore must needs make some little alteration in the Product.

Ex. IV. Suppose the length of a Rhomboids be 17 Foot 3 Inches, or 207 Inches, and the true breadth or line 8 Foot 7 Inches, or 91 Inches, what's the Content.



Vulgarly.

1.

207 length.

91 breadth.

$$\begin{array}{r}
 \text{F. I.} \\
 144) 18837 \quad (130.177 \text{ A.} \\
 \underline{443} \\
 117
 \end{array}$$

$$\begin{array}{r}
 207 \\
 \underline{1862} \\
 18837 \text{ A. I.}
 \end{array}$$

Decimally.

F. I.

17 25

7. 58

13800

8625

12075

130.7550 A. F. P.

The Reason of Measuring any of the preceding Figures, is shown in the Use of Theo. 7.

More

More Examples.

Suppose the side of a square be 18 Foot, 11 Inches $\frac{1}{2}$, what's the Area? *Answer* 359 Foot, 64 square Inches $\frac{1}{4}$.

Suppose the length of a Parallelogram or long Square be 24 Foot 5 Inches $\frac{1}{4}$, and its breadth 17 Foot 13 Inches, what's the Area? *Answer*. 437 Foot, 120 square Inches, 12 Quarters.

Suppose the length of a Rhombus be 35 Foot, 3 Inches $\frac{3}{4}$, and its true breadth 18 Foot, 4 Inches $\frac{1}{2}$, what's its Area? *Answer*. 648 Foot, 124 square Inches, 14 Quarters.

Suppose the side of a Rhomboids be 19 Foot, and its breadth 7 Foot, what's the Area? *Answer* 133 Foot.

P R O P. II.

To Measure a Triangle.

Rule 1. *Multiply the whole Base by half the Perpendicular; or the whole Perpendicular by half the Base; it gives you the Area, Or if you multiply the whole Base by the whole Perpendicular, and take half that Product, it gives you the Area.*

Example 1. Suppose the Base of a Triangle be 34 Foot 8 Inches, or 416 Inches, and the Perpendicular A B 15 Foot, 10 Inches, or 190 Inches, what's the Area?



Vulgarly.

(114)

Vulgarly.

I.

416 Base.

95 half Perp.

2080

3744

39520 A. I.

F. I.

44) 39520 (274. 64. A.

1072

640

64

Decimally.

F. P.

34. 67

7. 95

3467

31203

24269

274.2397 A. F. P.

If the Perpendicular be not given, it may be found having the three sides, by *Prop. 12* of the *Second Book of Euclid*.

The reason of measuring a Triangle by any of these Rules, is shown in the use of Theo. 8.

Rule 2. But in case you would find the Area without the Perpendicular, by having the three sides, do thus; Add the three sides together and half the Sum, then subtrall each side from that half Sum, and multiply this half Sum by each of the Remainders continually, and the the square Root of this last Product is the Area of the Triangle.

Ex.

Ex. II. Suppose the three sides of the Triangle are as in the annexed Figure in the Margin, what's the Area.

35 F. }
28 F. } the sides.
21 F. }

84 F. the Sum.

42 F. half Sum

7 diff. 1st side

14 diff. 2d side

21 diff. 3d side

42 half Sum

7

294

14

1176

294

4116

21

4116

8232

86436



) 86436 (294 A.F.

49) 464

584) 2336

PROP. III.

To Measure a Trapezium.

Rule. Divide it into two Triangles by a Diagonal, and drop Perpendiculars thereto, then multiply the whole Base by half the Sum (or half the Base by the whole Sum) of the Perpendiculars; the Product is the Area.

Ex.

Example 1. Suppose the Dimensions of a Trapezium are as in the Margin, what's the Area?



Vulgarly.

I.

150 } the Perpendiculars.
116 }

266 their Sum

133 half Sum

200 the Base.

26600 A. I.

F. I.

144) 26600 (184. 104. A.

1220

680

104

Decimally.

F. P.

16. 67 Base.

11. 08 half Sum Perp.

13336

16670

1667

184.7036 A. F. P.

Note, If two sides of a Trapezium are parallel, Add them together, and halve the sum; this half multiply by the nearest distance betwixt those two sides, the Product gives the Content. Or if you measure it in the middle betwixt those two parallel sides, it will be the same.

Ex.

Ex. The Dimensions of a Trapezium whose opposite sides are Parallel, are as in the Margin, what's the Content ?

F.	
22	} sides
18	
40	Sum
20	half Sum
12	Breadth
240	A. F.



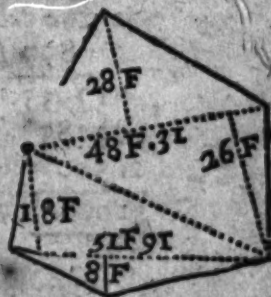
PROP. IV.

To Measure an Irregular Figure.

Rule. This and the like Figures are practically measured by dividing it into Triangles or Trapeziums; which done, *Measure each Triangle severally, and add the several Contents together, so will the Sum be the Contents of the said Figure.*

Examp. Suppose the Dimensions of an Irregular Figure are as in the Margin, what's the Area ?

Work according to the preceding Direction, and you shall find 2375 Foot, 6 Inches.



The reason of Operation, both in this and the preceding Proposition, is the same of that with Measuring the Triangle, they being all reduced to Triangles.

I

PROP.

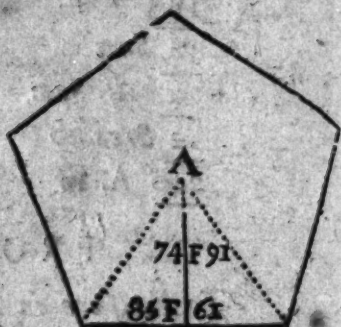
parallel,
is half
of two
if you
parallel
Ex.

PROP. V.

To Measure a Regular Polygon.

Rule. *Multiply the half Sum of all the sides, by the length of a Line drawn from the Center to the middle of any one side, the Product gives the Area.*

Ex. Suppose the side of a Regular Polygon be 85 Foot 6 Inches, or 1026 Inches. The Line AB drawn from the Center to the middle of one side 74 Foot 9 Inches, or 897 Inches, what's the Area?

*Vulgarly.*

I.

1026 side.

5 number of sides.5130 Sum of the sides.

2565 half Sum.

897 Line from the Center.

17955

23085

205202200805 A. L.

F. I.

144) 2200805 (15977. 117. A.

860140811201125117

Decimally.

Decimally.

F. P.

85.5

5

427.5

213.75

74.75

106875

149625

85500

14962515977.8125 A. F. P.

The reason of Measuring any Regular Polygon is very evident, for so many sides as there is in a Polygon, so many Triangles doth it contain, the Bases of which are the Polygons sides; and therefore if we add all the sides (or Bases of the Triangles together) and multiply half the Sum by the Line drawn from the Center to the Circumference, it gives the Area of the Polygon, that is, of all the Triangles at once.

More Examples.

Suppose the side of a Regular Hexagon be 48 Foot, and Line drawn from the Center to the middle of any one side 42 Foot, what's its Area? *Ans.* 6048 Foot.

Suppose the side of a Heptagon be 72 Foot, and Line drawn from the Center to the middle of any one side 75 Foot, what's its Area? *Ans.* 18900 Foot.

PROP. VI.

To Measure a Circle.

Before I give you the Rule for Measuring this Figure, it will be necessary to shew how, when the Diameter is given, to find the Circumference. & *contra*.

Proportions.

1 : 3.14159 or 113 : 355 :: Diam. Cir. *apply'd*
 1 : 0.31831 :: Circum. Diam. 149
 1 : 0.78540 :: Diam. Square. Area.
 1 : 1.27324 :: Area Diam. squar'd.
 1 : 0.70711 :: Diam. Side squ. inf.
 1 : 0.88623 :: Diam. Side squ. equ.
 1 : 0.07957 :: Cir. squ. Area.

By these Proportions may several Questions relating to the Circle be solv'd, and that only by Multiplication, For I have not given those common Proportions of 7 to 22 and the like, which requires both Multiplication and Division, but have put them in such terms, where the first is always 1 or unity, by which means I exclude the troublesom work of Division.

As *First*, Diameter 86 given to find the Circumference, multiply the given Diameter by 3.14159 and cut five Figures from the Product, it gives 270.17674 the Circumference.

And here Note, That it is at pleasure to use all the Decimals, for 3.14 will be near enough for any common use, but if greater exactness be required, you must use more Decimal Places.

Secondly,

Secondly, Circumference 156 given to find the Diameter, multiply the given Circumference by .31831 it gives 49.65636 the Diameter.

Thirdly, Diameter 24 given to find the Area, square the given Diameter, that is multiply it by it self, and that Product by .7854 it gives (452.1904) the Circles Area.

Fourthly, Area (600) of a Circle given to find its Diameter, multiply the Area by 1.27234 and extract the square Root of the Product, it gives (27.63) the Diameter.

Fifthly, Diameter given to find the side of the square inscribed, Multiply the given Diameter by 70711, the Product is the side of the square inscrib'd.

Sixthly, Diameter given to find the side of the square equal. Multiply the given Diameter by .88623, the Product is the side of the Square.

Seventhly, Circumference given to find the Area, square the Circumference, and multiply the Product by .07957 it gives the Area.

Note, These Tabular Numbers are raised and calculated after the same manner as the Tables of Sines, Tangents, &c.

Rule. Multiply half the Circumference by half the Diameter, it gives the Area of the Circle.

Ex. Suppose the Diameter of a Circle be 16 Foot 3 Inches, or 195 Inches, what's the Area.

(122)

Vulgarly

1.

195 Diam.

3.14

780

195

585

612.30 Cir.

306.15 half Cir.

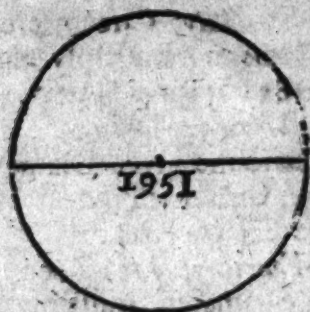
97.5 half diam.

153075

214305

275535

29849.625 A. I. P.



F. I.

144) 29849 (207.41. A.

1049

41

Decimally.

F. P.

16. 25

3. 14

6500

1625

4875

51.0250 Cir.

25.5125 half Cir.

8125 half Diam.

1275625

510250

255125

2042000

207.3890625 A.F.P.

Here

Here you see we must use Decimals in this Case, even when we do it vulgarly, and therefore a Circle or its part is easiest and quickest, measured all by Decimals.

More Examples.

The Diameter of a Circle is 54 Foot 2 Inches, what's the Area. *Ans.* 2304 Foot 44 Inches.

The Circumference of a Circle is 97 Foot 8 Inches, what's it's Area. *Ans.* 761 Foot 91 Inches.

PROP. VII.

To measure a Sector.

Rule. Multiply one of its Legs by half the length of the Arch Line, (or half the Leg by the whole Arch) the Product is the Content.

Exam. Suppose the Leg of a Sector be 18 Foot 6 Inches, or 222 Inches, and the length of the Arch-line 13 Foot 3 Inches, or 159 Inches what's it's Area.

Vulgarly.

$$\begin{array}{r}
 \text{I.} \\
 222 \text{ Leg.} \\
 111 \text{ half Leg.} \\
 159 \text{ Arch.} \\
 \hline
 999 \\
 555 \\
 111 \\
 \hline
 17642 \text{ A. L.}
 \end{array}$$



$$\begin{array}{r}
 \text{F. I.} \\
 144) \underline{17649} \quad (122. 81. \text{ A.} \\
 \underline{324} \\
 369 \\
 \underline{81}
 \end{array}$$

I 4

De

Decimally.

F. P.

13. 25

9. 25

6625

2650

11925

1225625 A. F. P.*More Examples.*

The reason of Measuring both the whole Circle and Sector, is shown in the case of Theo. the 6th, they being made up of an infinite number of Triangles.

The Leg of a Sector is 41 Foot 12 Inches, and the Arch 38 Foot 5 Inches, what's its Area. *Ans.* 797 Foot 21 Inches.

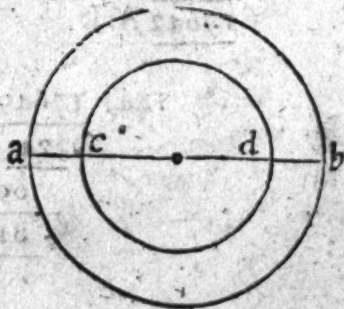
The Leg of a Sector is 205 Foot, and the Arch 23 Foot 9 Inches, what's the Content. *Ans.* 2434 Foot 54 Inches.

Note, If you would find the Area of a Semicircle, Quadrant, Sextant, &c. or any other part of a Semicircle, 'tis but finding the Area of the whole Circle, and then dividing it by 2, 4, 6, &c. the Quotient gives the Area of the part required.

P R O P. VIII.

To measure a Parallel Segment of a Circle.

Rule. Find the Area of the greater Circle, as also the Area of the lesser, Subtract the Area of the greater Circle from that of the lesser, the Remainder is the Area of the Segment sought.



Ex.

Ex. Suppose $a b$ the Diameter of the greater Circle be 57 Foot, and $c d$ the Diameter of the lesser Circle 43 Foot, what's the Area of the intercepted *Annula* or Ring.

Work, and you shall find .1099 Feet and 49 Inches

P R O P. IX.

To measure the Segment of a Cir. cut off by a Chord line.

Rule. From the Segment make a Sector, then the reason of measuring the Segment of a Circle is evident; For find the Area of the Sector, and also the Area of the Triangle, then deduct the Area of the Triangle from the Area of the Sector, the Remainder is the Area of the Segment.

Ex. Suppose the length of a Segment Arch $a o b$ be 31 Foot, and the Chord $a b$ 26 Foot, what's the Area.

F.

31 Arch

15.5 half Arch

19 Leg

1395

155

2945 a Sector

182 a Triangle

112 a Segment

26 Chord $a b$

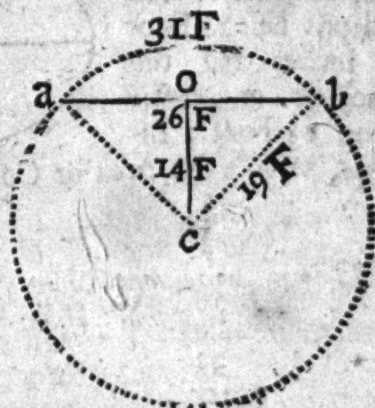
13 half Chord

14 Perpendicular

52

13

182 a Triangle.



P R O P.

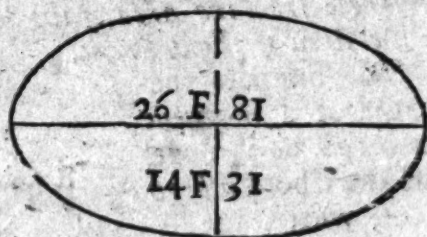
PROP. X.

To measure an Oval or Ellipsis.

Def. An Oval or Ellipsis is a Figure bounded by one Curve Line, generated from the Section of a Cone, by a Plain passing through both sides.

Rule. Multiply the longest Diameter by the shortest, and Extract the Square Root of the Product, it gives the Diameter of a Circle, whose Area is equal to the Area of the Ellipsis.

Ex. Suppose the longest or Transverse Diameter of an Ellipsis be 26 Foot 8 Inches and the shortest or conjugate 14 Foot 3 Inches, what's the Area or Content.



I

320 long Diam.

171 short Diam.

320

2240

320

) 54720 (233 Diam. Circle equal.

43) 147

463) 1820

431

(127)

I.

233

233

699

699

466

As 1 to 4785 so is 54289 Sq. of the Diam.

.785

271445

434312

380023

42616865 Area of the Circle

equal to the oval in I.

F. I.

44) 41831 (295. 136. A. of the Elipsis.

1381

856

More Examples.

Suppose the longest Diameter of an Elipsis is 140 Foot, and the shortest Diameter 87 Foot, what's the Area. *Answ.* 9544 Foot 109 Inches.

Suppose the longest Diameter of an Elipsis be 86 Foot 2 Inches, and the shortest 59 Foot 1 Inch, what's the Content. *Answ.* 3395 Foot 101 Inches.

PROP. XI.

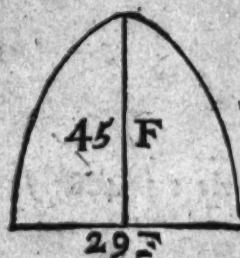
Def. A Parabola is a Figure bounded by two Lines, one of which is curve, the other straight; it is generated by cutting a Cone with a plain Parallel to the side thereof, as the Figure.

I.

To Measure a Parabola.

Rule. Multiply the line by the line and that Product again by 2, Divide this last Product by 3, the Quote is the Area of the Parabola.

Ex. Suppose the breadth of the Adjacent Parabola be 29 Foot, and the Height 45 Foot, what's the Area.



$$\begin{array}{r}
 \text{F.} \\
 29 \\
 \times 45 \\
 \hline
 145 \\
 116 \\
 \hline
 1305 \\
 \times 2 \\
 \hline
 2610
 \end{array}
 \qquad
 3) \begin{array}{r} 2610 \\ \hline 21 \end{array} (870 \text{ F. A.}$$

The Demonstration of the reason of Measuring the two last Figures, is founded on the Doctrine of the Conick Sections; where 'tis proved, that a mean Proportional betwixt the longest and shortest Diameter of an Ellipsis, is the Diameter of a Circle, whose Area is equal to that of the Ellipsis; and that the Parabola is two thirds of its circumscribing Parallelogram.

And here Note, that the best way for practically taking the Dimensions of any of the forementioned Figures, whether upon the Ground, Walls or Ceilings, is either by a ten or twelve Foot Rod, or by a Line stretch't from Corner to Corner; For Perpendiculars, they are actually dropt or rais'd by a line applied to one side of a Square.

THE

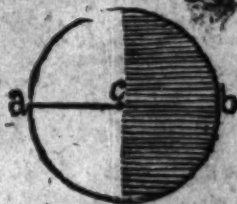
The Measuring of SOLIDS.

PROP. XII.

To Measure a Globe or Sphere.

Rule. First find the Circumference, which multiply by the Diameter, the Product is the Superficial Content; this Superficial Content, multiplied by $\frac{1}{2}$ of the Diameter, the Product is the Solid Content.

Ex. Suppose *ab* the Diameter of a Sphere is 2 Foot 6 Inches, what's the Solid Content.

*Decimally.*

3.141

2.5 Diam.

15705

6282

78525 Circum.

2.5

392625

157050

19.63125 Superficial Content.

.417 $\frac{1}{2}$ of the Diam.

13741875

1963125

7852500

8.18623125 Solid Content in F. & P.*Vulgarly.*

Vulgarly.

3.141

30 Diam. in l.

94.230 Circum in l.

20

2826.900 A. of its Surface

5 $\frac{1}{4}$ of the Diam.14134 900 Solid Content of the Sphere in l.

1728) 14134 (8 F. 310 l. Content of the Sphere

310*More Examples.*

Suppose the Diameter of a Sphere is 63 Foot
what's the Solid Content. *Answer* 130924 Foot
1036 Inches.

Suppose the Diameter of a Sphere is 86 Foot
Inches, what's the Solid Content. *Answer* 33888
Foot 998 Inches.

Suppose the Circumference of a Sphere is 8 Foot
6 Inches, what's the Solid Content. *Answer* 11
Foot 767 Inches.

Notes.

The Content of any Solid being found in Inches
Divided by 1728 the Solid Inches in one Foot,
gives the Content in Solid Feet, if the Inches that
remain is half or a Quarter of 1728, then its half
or a Quarter of a Solid Foot.

Secondly, If the Circumference of a Sphere be
given, then find the Diameter, as before has been
Taught.

Thirdly. In the vulgar way of Measuring this
and other of the following Solids, that in Multiplying
in

ing or Dividing we reject the Decimals, which arise by finding the Circumference or Diameter.

The Reason of Multiplying the Circumference by the Diameter, for finding the Superficial Content, is because the curve Surface of the Circumscribed Cylinder is equal to the curve Surface of the Sphere: And the reason of finding the Solid Content, is evident from Theo. 44.

PROP. XIII.

To Measure a Spheroid.

Def. A Spheroid is a Solid Oval, generated by the motion of an Ellipsis, upon its Transverse or longest Diameter, being in shape like a Pumkin, as the Figure A B C D. For Measuring of which this is the

Rule, By the Circumference and Diameter find the Area of the lesser Circle; this Area multiply by $\frac{2}{3}$ of the longest Diameter, the Product is the Solid Content.

Exam. Suppose the shortest Diameter of a Spheroid be 6 Foot 7 Inches, and the longest 9 Foot 9 Inches, what's the Content.



Vulgarly

Vulgarly.

$$\begin{array}{r} 3.141 \\ \underline{79} \text{ I.} \end{array}$$

$$\begin{array}{r} 28269 \\ \underline{21987} \end{array}$$

$$\begin{array}{r} 248.139 \text{ Circum.} \\ \underline{124.069} \text{ half Circum.} \end{array}$$

$$\begin{array}{r} 395 \text{ half Diam.} \\ \underline{620345} \end{array}$$

$$\begin{array}{r} 1116621 \\ \underline{372207} \end{array}$$

$$\begin{array}{r} 4900-7255 \text{ A. I.} \\ \underline{78} \text{ two thirds of the long Diam.} \end{array}$$

$$\begin{array}{r} 392058040 \\ \underline{343050785} \end{array}$$

$$\begin{array}{r} 382256.5890 \text{ Content in I.} \end{array}$$

F. I.

1728) 382256 (221. 368 Solid Content of the Spheroid.

$$\begin{array}{r} 3665 \\ \underline{2096} \end{array}$$

$$\begin{array}{r} 368 \end{array}$$
Decimally.

(133)

Decimally.

6.583

3.141

6583

26332

6583

19749

20677203 *Circum.*

10338601 *half Cir.*

3291

10338601

93047409

20677202

31015803

34024335891

65

170121679455

204146015346

221.1581832915 *Solid Cont. in F. P.*

Another Example.

Suppose the longest Diameter of a Spheriod be 34 Foot, and the shortest Diameter 18 Foot, what's the Solid Content. *Ans.* 4071 F and $\frac{1}{2}$.

The reason of Measuring this Figure depends upon the Knowledge of Doctrine of Conick Sections.

PROP. XIV.

To Measure all sorts of Prisms, as Parallelepipeds, Cubes, Cylanders, &c.

Under this Figure (as before intimated) is comprehended the Cube, Parallelepipedon, Cylander and all other Solids, that are as big at one end as the other; For Measuring which this is the

K

Rule:

Rule. Find the Area of the Base, whether it be Triangle, Square, Circle, or whatever else, this Area multiply by the length or height of the Prism, and the Product gives you the solid Content.

Ex. I. Suppose the breadth of a Parallelepipedon L be 3 Foot 9 Inches, its thickness 2 Foot 3 Inches, and its length 18 Foot 6 Inches, what's the Solid Content.



Vulgarly.

45 breadth in l.

27 thickness in l.

315

90

1215

222 Length in l

2430

2430

2430

1728) 269730 (156 F. 162 l solid Con

.9693

10530

162

Decimally. 275

2.25

1875

750

750

8.4375

18.5

421875

675000

84375

156.09375

The Cube is also Measured by this Rule; and therefore, since its length, breadth and thickness are equal, 'tis but Cubeing of its side; *That is*, Multiplying one side three times into its self, the Product is the solid Content.

Ex. 2. Suppose the side of a Cube be 7 Foot, what's the solid Content.

F.

$$\begin{array}{r} 7 \\ \times 7 \\ \hline 49 \\ \times 7 \\ \hline \end{array}$$

343 F. Solid Cont.



Ex. 3. Suppose the Diameter of the Base of a Cylinder be 6 Foot 9 Inches, and the Perpendicular height 12 Foot 3 Inches, What's the solid Content.

Vulgarly 3.141

81 Diam. in I.

$$\begin{array}{r} 3141 \\ \times 81 \\ \hline \end{array}$$

251281 Circum.

127.210 half Circum.

81

127210

1017680

10304.010

5152.005 Area of the Base.

✓ 47457 height in I.

36064035

29608020

5152005

757344.735 Content in I.

1728) 757344 (438 F. 480 I. Solid Content of the Cylinder.

.6614

14304

.480

K 2

Decimally.



lid Con

The

(176)

Decimally.

3.441

6.75

15705

21987

18846

21.20175

10.60087

3.38

8480696

3180261

3180261

35.8309406

12.25

1791547030

716618812

716618812

358309406

438.929022350

Solid Cont. in F. P.

Exam. IV. Suppose the side of an Equilateral, or a triangular Prism be 2 F. 6 I. and its length or height 23 F. 9 I. What's the solid Content.



Vulgarly.

(117)

Vulgarly.

30 } the 3 sides in I,
30
30

90 Sum in I

45 half Sum

15 Differ.

225

45

675

15

3375

675

10125

15

50625

10125

) 151875 (389 Root.

68) 618 285 height I.

769) 7475 (1945

554 3112

778

110805 Cont. I.

1728) 110865 (64 F. 273 I. Solid Content.

.7185

273

K 3

Deci.

(138)

Decimally.

$$\begin{array}{r} 2.5 \\ 2.5 \\ 2.5 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{The 3 sides.}$$

7.5 Sum.

3.75 half Sum.

1.25

1875

750

375

46875

1.25

234375

93750

46875

5.859375

1.25

29296875

11718750

5859375

7.32421875 (2.7063 Area of the Base

47) 332

23.75

5406) 34218

135315

54123) 178275

189441

15906

81189

54126

64.270625 Solid Cont.

More Examples.

Suppose the breadth of a Parallelepipedon is 22 F. 6 I. and the thickness 14 F. 3 I. and the height 32 F. 9 I. what's the solid Content. *Ans.* 10500 F. 810 I. Suppose

Suppose the Diameter of a Cylinder is 46 F. 7 I. and the Perpendicular height 124 F. 5 I. What's the solid Content: *Ans.* 212005 F. 1079 I.

Suppose the side of a Cube is 16 F. 9 I. what's the solid Content. *Answer.* 4699 F. 729 I.

Suppose the side of an Equilateral Triangular Prism is 3 F. 10 I. and the length 37 F. what's the Solid Content. *Answer* 235 F. 624 I.

After the same manner is the Content of all Polygonous Prisms found, that is, such whose Bases contain 5, 6, 7 or 8 sides.

PROP. XV.

To Measure a Cone or Pyramid.

Rule. First find the Area of the Base, whether it be Triangular, Quadrangular or Circular, which is the Base of the Cone; then multiply this Area by $\frac{1}{3}$ of the Perpendicular height, the Product is the solid Content.

Ex. I. Suppose the side of a square Pyramid be 2 Foot 8 Inches, and the Perpendicular height 24 Foot 9 Inches, what is the solid Content.



(140)

30 } Side of the Square in I.
30 }

900 Area of the Base.

99 $\frac{2}{3}$ of the height in I.

8100

8100

89100 Solid Content in I.

1728) 89100 (51 F. 972 I. Solid Content.

2700

972

Decimally.

2. 5

2. 5

125

50

625

825

3125

1250

5000

51.5625 F. P. Solid Content.

Exam. II. Suppose the Diameter of a Cone be 4 Foot 6 Inches, and the Perpendicular height 18 Foot 9 Inches, what's the Solid Content.



Vulgarly.

(141)

Vulgarly. 3.1414

54 Inches Diameter.

125656

157070

169.6356 Cir. L.

84.82

27

59374

16964

2290.14 A. of the Base in L.

77 L. $\frac{1}{4}$ of the height

16 03098

1603098 F. L.

1728) 171634078 (99.688. Sol. Cont.

16240

688

Decimally. 3.141

4.5

15705

12564

14.1345

7.0672

2.25

353360

41. 344

141344

15.901200

6.25

79506000

31802400

95407200

99.3825

A. F. P;

Note.

Note. If you would find the Area of the Curve Surface of a Cone, 'tis done by multiplying the whole Circumference by half the slant height, or half the Circumference by the whole slant height; the Product gives the Solid Content.

For Example. Suppose as in the preceeding Cone, the Circumference be 166 Inches, and the slant height 18 Foot 9 Inches, what's the Area of the Curve Surface.

$$\begin{array}{r}
 166 \text{ Circumference in l.} \\
 83 \text{ half Circumference,} \\
 \hline
 225 \text{ slant height in l.} \\
 \hline
 415 \\
 166 \\
 \hline
 166 \\
 \hline
 18675 \text{ A. in l.} \\
 \hline
 \text{F. I.} \\
 144). 18675 \text{ (129. 99. A,} \\
 \hline
 427 \\
 \hline
 1395 \\
 \hline
 99
 \end{array}$$

More Examples.

Suppose the longest side of the Base of an oblong Pyramid be 42 Foot 9 Inches, and the shortest side thereof 16 Foot 4 Inches, and the Perpendicular height 63 Foot 3 Inches, what's the Solid Content. *Answer* 14721 Foot 756 Inches.

Suppose the Diameter of the Base of a Cone be 26 Foot 6 Inches, and the Perpendicular height 48 Foot 9 Inches, what's the solid Content. *Answer* 17082 Foot.

Sup.

Suppose the Circumference of the Base of a Cone be 78 Foot, and the slant height 96 Foot, what's the Area of the Curve Surface of that Cone. *Answer* 3744 Foot.

PROP. XVI.

To Measure the Frustrum of a Pyramid or Cone.]

Def. The Frustrum of a Pyramid or Cone is only the remaining part when the Top is cut off, such are the Figures X and Y in the following Examples.

Rule. Find the Area of both Bases, then multiply the one by the other, and extract the square Root of the Product; This square Root add to the Area of both Bases, the Sum multiply by $\frac{1}{3}$ of the Perpendicular height, the Product is the Solid Content.

Exam. 1. Suppose the side of the greater Base of the Frustrum of a true Square Pyramid be 3 Foot 6 Inches, or 42 Inches the side of the lesser Base 2 Foot 3 Inches, or 27 Inches, and the Perpendicular height 6 Foot 9 Inches, or 81 Inches, what's the Solid Content.



side

(144)

$\begin{array}{r} 27 \\ 27 \end{array} \}$ side lesser Base.

189

54

729 A. lesser Base.

Vulgarly.

$\begin{array}{r} 42 \\ 42 \end{array} \}$ side greater.

84

168

1764 A. greater Base.

729 A. lesser Base.

15876

3528

12348

1285956 A. of both Bases.

) 1285956 (1134

21) 28 1764 A. great Base.

223) 759 729 A. less Base.

3627

2264) 9056 27 * of the heights.

25389

7254

97929 Solid Content in I.

1728) 97929 (56 F. 1161 I. Solid Content.

11529

1161



Decimally.

Decimally.

$$\begin{array}{r}
 3.5 \quad 2.25 \\
 \underline{3.5} \quad \underline{2.25} \\
 175 \quad 1125 \\
 \underline{105} \quad \underline{450} \\
 12.25 \quad 450
 \end{array}$$

$$\begin{array}{r}
 5.0625 \\
 \underline{12.25} \\
 253125 \\
 101250 \\
 101250 \\
 50625 \\
 \hline
 62015625
 \end{array}$$

$$62015625 \cdot (7.875)$$

$$\begin{array}{r}
 148) 1301 \quad 12.25 \\
 \underline{1567} \quad \underline{11756} \quad \underline{5.0625}
 \end{array}$$

$$\begin{array}{r}
 45745) 78725 \quad 25.1875 \\
 \underline{} \quad \underline{2.25}
 \end{array}$$

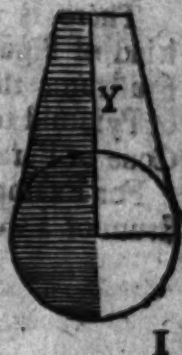
$$1259375$$

$$503750$$

$$503750$$

$$56.671875 \text{ F. P. Sol. Cont.}$$

Exam. II. Suppose the Diameter of the greater Base of a Cone be 19 Inches, and of the lesser Base 14 Inches, and Perpendicular height 48 Inches, what's the solid Content.



I. P.

283.52 A. of the greater Base.

153.93 A. of the lesser Base,

43642.2336 Product of both Bases.208.9 Sq. $\sqrt$ of the Product of both Bases.

283.52

153.93

646.35

16 a third of the height.

387810

646351034160

1728) 10341 (5 F. 1701 I. Solid Content.

1701

And here note that not only in this, but in several of the preceeding and following Examples, we have rejected part or all the Decimal Places, when we come to multiply or divide them by other Numbers; for which reason several of the Area's are not exact, tho' near enough for common Business.

For the Reason of the Rule given for measuring these Frustrums. See Oughtred's Clavis. Page 141, &c.

There is another Method for measuring these kind of Figures, which is done by compleating the Frustrum; that is, by adding another perfect Cone, or Pyramid to it, and so make the Frustrum a Perfect Cone or Pyramid it self.

For Example, let the Figure *a b f n e* be the Frustrum of a Cone.

Now

Now it is evident, that if by Prob. 14 I find the solid Content of the Whole Cone, and also the Content of the added Cone $en f$, and then subtract the Area of the lesser Cone from the Area of the greater, the Remainder is the Area of the Frustrum $abfn e$.

The method of Compleating the Frustrum, is thus,

As $a c$ the half difference of the Diameters to $c e$ the height of the Frustrum, so is $e r$ the half Diameter of the lesser Base, to $r n$ the height of the Whole Cone, by *Theo.* 18.

Having thus obtained the height of the whole Cone, deduct the height of the Frustrum therefrom, and the Remainder is the height of the little Cone that must be added.

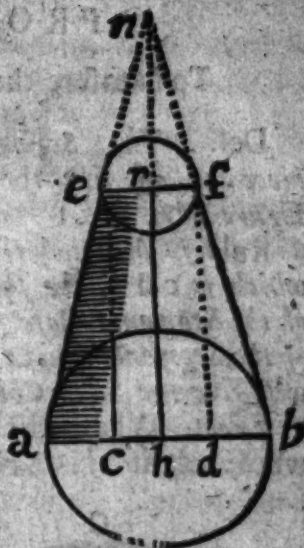
This Proportion will hold good for Square Pyramids, if instead of the half difference of the Diameters, you use the half difference of the sides.

More Examples.

* There is the Frustrum of a Pyramid whose two Bases are Oblongs, the greater Base has its longest side 21 Foot 6 Inches, the short side 14 Foot 4 Inches; the lesser Base has its long side 9 Foot 6 Inches, and its shortest side 6 Foot, 4 Inches, and the Perpendicular height is 36 Foot, what's the solid Content? *Ans w.* 6054 F.

There is the Frustrum of a Cone, the Diameter of the greater Base is 16 Foot, the Diameter of the lesser is 14 Foot, and the Perpendicular height is 24 Foot, What's the Solid Content. *Ans w.* 4248 F.

PROP.



P R O P. XVII.

To Measure the Segment of a Sphere.

Def. The Segment of a sphere or Globe, is a piece cut off like the slice of an Apple, such is the Figure *a b c d*.

Rule. Multiply the treble height of the Segment, which is *cd* by the square of the Radius *ad* or *db* of the Segments Base, adding to the Product the Cube of the Segments height: The Sum multiplied by .5235 will give the Solid Content of the Segment.

Exam. What's the solid Content of the Segment, whose height *cd* is 3 Foot, and Diameter *ab* 12 Foot.



$$\begin{array}{r}
 6 \frac{1}{2} \text{ Radius.} \\
 \hline
 36 \text{ sq. of Radius.} \\
 9 \text{ Treb. height.} \\
 \hline
 324 \\
 27 \text{ Cube of the height} \\
 \hline
 351 \\
 5235 \\
 \hline
 1755 \\
 1053 \\
 702 \\
 1755 \\
 \hline
 182.7485 \text{ F. Solid Content.}
 \end{array}$$

If you find the Area of its Curve Surface, then Multiply the Circumference of the Base by the Altitude of the Segment; The Reason of which is, because the curve Surface of the Circumscribing Cylinder is equal to the Curve Surface of the Segment.

Another

Another Example.

Suppose the height of a Segment of a Sphere be 24 Inches, and the Radius of its Base 68 Inches, what's it's solid Content. *Ans.* 105 F. 84 I.

P R O P. XVIII.

To measure the middle Frustrum of a Spheroid, in which form are most of our common Beer and Ale Casks.

Rule. Find out the Area's of the two Circles belonging to the Diameter at Head and Bung, and then add to $\frac{1}{4}$ of the lesser Area $\frac{3}{4}$ of the greater Area, the Sum multiply by the length of the Cask, the Product is the Content in Cubick Inches; which if you divide by 282 gives the Content in Ale Gallons, or by 231 gives the Content of the said Cask in Wine Gallons.

Exam. Suppose eb the head Diameter of a Cask be 24 Inches, ab the Bung Diameter 34 Inches, and its length cd 48 Inches, how many Ale Gallons doth it contain.

Vulgarly.

See page 120

3.1415

24 Diam. at Bung.

125660

94225

106.8110

53.4055

17

3738385

534055

9078935

605.2623

L

multiplied by 34 instead of 24

Cir.

half Cir.

half Diam. Bung

A. of the Cir. at Bung.

two thirds.

3.1415

(150)

3.1415

24 Diam. at Head.

125660

62830

75.3960 Cir.

37.698 half Cir.

12 half Diam.

451.376 A. of the Cir. at head,

159.458 One third

605.6262 two thirds.

150.458 one third.

756.0842

48 length

60486736

30243368

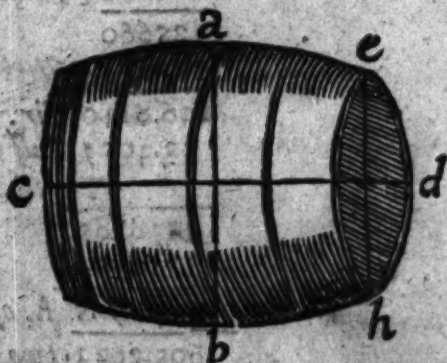
36292.0416 Content in Cube Inches.

282) 36292 (128 Ale Gallons.

809

2452

196



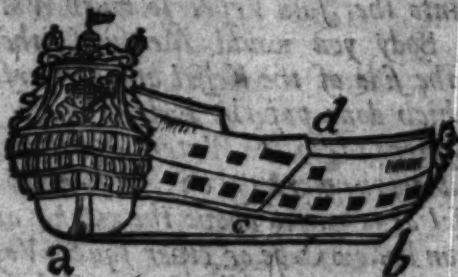
PROP.

PROP. XIX.

To measure a Ship, *that is*, to find her Tunnage

Rule. Multiply the breadth of the Ship, that is, the length of the Midship Beam by the half breadth, and that Product again by the length of the Keel it gives her solidity in Cubick Feet; then divide this last Product by 94, the Quote will give you the Tunnage of the Ship.

Exam. Suppose a Ships Keel as *ab* is 84 Foot, and the breadth or length of the Beam *cd* 26 Foot, what's her Content.



$$\begin{array}{r}
 26 \text{ breadth.} \\
 13 \text{ half breadth} \\
 \hline
 78 \\
 26 \\
 \hline
 338 \\
 84 \text{ length of the Keel.} \\
 \hline
 1352 \\
 2704 \\
 \hline
 94) 28392 \text{ (302 Tuns.)} \\
 \hline
 0192 \\
 \hline
 04
 \end{array}$$

L 2

Others

Others have this Rule for Measuring a Ship.

Measure the length of the Keel, Midship Beam and Depth of the Hold, these three numbers Multiply one by the other, Divide the Product by 95, and the Quote will give the true Burthen of any Merchants Ship: For Men of War divide by 100, because in them there is an Allowance made for Ordnance, Masts and Sails, &c.

PROP. XX.

To find the Content of any Irregular Solid, whether Lead, Iron, Stone, Wood, &c.

Rule. To effect this, take a convenient Vessel, and pour into the said Vessel so much Water as will cover the Body you would Measure, then make a mark on the side of the Vessel at the Surface of the Water; which done put the Body, (whose solid Content is required) into the Water, and it will cause it to rise up above the mark; then take so much Water out of the Vessel as is raised above the mark, and put the same into an hollow Cube or other Square Vessel, which you ought to have in readiness for such a purpose; the solid Content of this Water in the Cube, is the solid Content of the Body required.

Some useful Proportions for Measuring of Solids.

As 1 : 0.523599 :: Diameter Cub'd : Solid. Sph.

As 1 : 0.016887 :: Periphery Cub'd : Solid. Sphr.

As 1 : 1.9086 :: Solid Spheroid to Cube Diameter,

As 1 : 0.523599 :: the square of the Conjugate Diameter of a Spheroid, multiplied into the transverse or long Diameter to the solid Content of the Spheroid.

As 1 : 0.785399 :: the square of the Diameter of the Base of a Cylinder, multiplied into its length,

to the solidity of the Cylinder, Or so is the square of the Diameter of the Base of a Cone, multiplied by $\frac{1}{3}$ of the Axis, to the solid Content of the Cone.

These Proportions are fram'd after the same manner as those in page 129. and their use to the same purpose, an Instance of which I shall here insert.

First, Diameter 24 of a Sphere given, to find the solid Content. Multiply the Cube of the given Diameter by .523599, it gives, rejecting the Decimals 7238 the solid Content of the Sphere.

Secondly, Circumference of a Sphere 24 given, to find the solid Content. Multiply the Cube of the given Circumference by .016887, it gives 42 the solid Content.

Thirdly, The Solidity of a Sphere 126 given to find its Diameter, multiply the given Solidity 126 by 1.9086, the Product gives 1240.4836 the Cube of the Diameter, the Cube Root of which is 132.25, the Diameter of the Sphere required.

Fourthly, Conjugate Diameter 4.1, and transverse 7.2 of a Spheroid given, to find the Solid Content. Multiply the square of the Conjugate by the Transverse Diameter, and that Product again by .523599, it gives 63 the solid Content.

Fifthly, The Base of the Diameter of a Cylinder is 12, and the height 18 given, to find the solid Content. Multiply the square of the Diameter of the Base by the Height, and the Product again by .785399, the Product 2035 is the solid Content.

Sixthly, If the Diameter of the Base and Height of a Cone are as in the preceeding Case of the Cylinder, then the solid Content of the Cone is 678 near.

Directions for the measuring Most Sorts of Artificers Works.

THE Principal Works that are generally measur'd, are Masons, Carpenters, Bricklayers, Plaisterers, Joiners, Glaziers, Painters and Paviers, several of which give their Contents by different Measures; *that is*, some in Rods, others in Squares, others in Yards and some in Feet, both Square and Running Measure.

By Running Measure is meant length without Breadth; *that is*, so many Foot long is so many Foot Running or Line Measure.

Now when we take the Dimensions of any kind of Work, we do it by Feet; *that is*, we say so many Feet and Inches by so many Feet and Inches, and from such Dimensions we cast up the Content in square Feet and Inches, after which we reduce it to square Rods, Yards, &c. according to the Measure the Work is measured by.

The Method used in setting down the Dimensions as we take them, and then casting them up, is as follows,

Dimensions.

F.	I.	P.
24.	10.	0
8.	9.	0
3.	5.	6
2.	8.	9.

The meaning of it is this, that 24 Foot 10 Inches is to be multiplied by 8 Foot 9 Inches; and 3 Foot 5 Inches and 6 Parts, is to be multiplied by 2 Foot 8 Inches and 9 Parts.

The Method of casting them up is thus, first multiply 24 Foot by 8 Foot, it gives 192 Foot; then

10 Inches by 8 Foot gives 80 Inches, which divide by 12 it gives 6 Foot 8 Inches; set this under 192 Foot. And here note that the 80 Inches are not square Inches, but such pieces that are an Inch broad and one Foot long, as is very evident from a little consideration: Then multiply the 24 Foot by 9 Inches, it gives 216 Inches, which divided by 12 gives 18 Foot, this you must set under the former 6 Foot 8 Inches: Lastly, multiply 10 Inches by 9 Inches, the Product is 90 parts, which divided by 12 gives 7 Inches 6 Parts; set this down also under the former in its proper Column. Having thus cast up these Dimensions, and set down the Content right against it, according to the preceeding Direction, add up each Row, and set down all above 12, carrying the Twelves to the next Row, just as you do in the Addition of Pence till you come to the last Row, which must be added up all as Integers.

After this manner is the Dimensions of all kinds of work both set down and cast up. See the Example following.

Dimensions.

F.	I.
24.	10.
8.	9.
192 :	
6 :	8
18 :	0
	7 : 06
217 :	03 : 06

By this you see that the Product of 24 Foot 10 Inches by 8 Foot 9 Inches, is 217 Foot 3 Inches, 6 Parts, this is set down against its Dimensions thus

L 4

F.

proceeding after this manner with $\begin{array}{r} \text{F. I.} \\ 24. 10. 7 \\ 8. 9. \end{array} \left. \vphantom{\begin{array}{r} \text{F. I.} \\ 24. 10. 7 \\ 8. 9. \end{array}} \right\} \text{F. I.P.} \quad 217.3.6$
 the rest, setting down the proper Products against
 their respective Dimensions, which being done, we
 cast up all the Products into one total Sum of Feet,
 and by Division turn it into square Rods, Yards, &c.

If you take your Dimensions nearer than Inches,
 as in some sort of Choice Work (particularly Gla-
 zing) you do, that is to Halves and Quarters, then
 you must for a quarter of an Inch set down 3 parts,
 and for half an Inch 6 Parts, and for $\frac{3}{4}$ of an Inch
 9 Parts, for we count 12 parts in an Inch, as well
 as 12 Inches in a Foot; after which cast up the Di-
 mensions, as in the following Example.

$$\begin{array}{r}
 \text{F. I. P.} \\
 3. 5. 6 \\
 2. 8. 9 \\
 \hline
 6. 10. \\
 1. \\
 2. 3. 4 \\
 4 \\
 2. 3 \\
 3. 9 \\
 4. 6 \\
 \hline
 9:05:03:01:06
 \end{array}$$

But that you may not mistake the Nature of the
 Product in multiplying several Dimensions together,
 I shall give you this general Rule.

Feet multiplied by Feet gives Feet.

Feet by Inches gives Inches.

Feet by Parts gives Parts.

Inches by Inches gives Parts.

Inches by Parts gives Seconds.

Parts by Parts gives Thirds.

Note,

Note, In casting up your Dimensions, some contractive ways may be used; for Instance, in the first of the preceeding Examples, instead of multiplying the 24 Feet by 9, and dividing by 12, take but $\frac{3}{4}$ of 24 and it performs the business: Also if you are to multiply any number of Feet by 6 Inches, take half the said Feet, and 'tis the same thing as if you had multiplied by 6 and divided by 12. In like manner to multiply any Number of Feet by 3 Inches, 'tis but taking $\frac{1}{4}$ of the Feet, and its the same thing as if you had multiplied by 3 and divided by 12.

And note, that if in any sort of Work there be two Dimensions alike, then is such Dimensions not set twice over, but thus.

4. 3 } twice.
2. 6 }

Having premised these general Notions, I shall now pass to the particular Methods and Customs used by Artificers, for Measuring each of their Works.

Instructions for Measuring of Brick-Work.

IF a Brick Wall be one Rod square and a Brick and half thick, then is such a piece of Brick Work a Rod of Brick Work, but if a piece of Wall be a Rod square, and more or less than one Brick and $\frac{1}{2}$ thick, then is the said piece of Wall accordingly more or less than one Rod of Brick work; And therefore if a Brick Wall be thicker or thinner than one Brick and $\frac{1}{2}$, it must be reduced to that thickness; for in a Piece of Brick work one Rod square and three Bricks thick, there is two Rods of Brick work, by reason 'tis double the thickness it should be; which work of Reduction makes the measuring of Brick work more troublesom than any other sort of Work. And

And here note, that by 1 B. $\frac{1}{2}$ is meant a Brick in length and another in breadth, that is, 9 Inches, and 4 Inches and $\frac{1}{2}$, which with a joint of Mortar makes 14 Inches, the length of one Brick and $\frac{1}{2}$.

This Work is generally measur'd by the Rod, *that is*, 'tis commonly brought into square Rod, which contains 272 Foot $\frac{1}{4}$, Or 272 Foot, 36 Inches, and is produced by Multiplying 16 Foot $\frac{1}{2}$ by 16 Foot $\frac{1}{2}$.

Now when you come to measure the Walls of a Building, some are thicker, others thinner than one Brick and $\frac{1}{2}$, which several thickneses must all be brought to the common thickness of one Brick and half, thus.

Take the Dimensions as they come in order, whether they be thicker or thinner, and set them down according to the following Method, with the thickness over each Dimension, then cast up the Content of each Dimension in square Feet and Inches, after which reduce it to half a Brick thick; thus if the Dimension is half a Brick thick, set it down as the Product gives it; if it be 1 Brick thick, multiply the Product of your Dimensions by 2; If the Product be one Brick and half multiply it by 3; if it be 2 Bricks thick multiply the Product by 4, &c.

This done, sum up all the Dimensions, it gives the total Sum in square Feet of half a Brick thick, which total divide by 3 will give the Content in Feet of 1 Brick and $\frac{1}{2}$ work, then if you divide the Content by 272, omitting the 36 odd Inches, the Quote will give you the Content in square Rod of Brick and half Work.

$\frac{1}{2}$ B. F. I. P.
$$\begin{array}{r} 8. 2 \\ 6. 5 \end{array} \} 52.4.10$$

$$3) 1737 \text{ (579 Foot of } B. \frac{1}{2} \text{ Work.)}$$

2 B.

272) 589 (2 Rod 45 Foot. Content

$$\begin{array}{r} 11. 4 \\ 2. 11 \end{array} \} 132.2.8.$$

45 of the said Dim.

3 B $\frac{1}{2}$

$$\begin{array}{r} 17. 9 \\ 8. 4 \end{array} \} 1035.5.0$$

The odd 11 Inches and 2
Parts we reject when we
Divide by 3.

1 B.

$$\begin{array}{r} 26. 4 \\ 9. 10 \end{array} \} 517.10.8$$

 1707.11.2

Some Measurers cast up their Dimensions thus, they multiply their Dimensions together, and set the Content of each Dimension against it in its proper thickness; *that is*, without reducing it to $\frac{1}{2}$ Brick work: Then they take out all the Contents of the same thickness and add them together; After which reduce them to $\frac{1}{2}$ Bricks, and then to Brick and $\frac{1}{2}$, which done, add them and the Brick and $\frac{1}{2}$ work all together; the Sum divided by 272, the Quote is Rods of Brick work in the whole at Brick and half thick; which of these two ways you like best, that use in Practice.

Notes.

First, In Brick work we seldom take the Dimensions nearer than Inches; *that is*, we do not measure to Halves or Quarters, unless in very fine Work.

Secondly,

Secondly, 'Tis evident that 272 Foot $\frac{1}{2}$ is a square Rod; but in dividing for Rod, we omit the Quarter, and divide by 272 only, the Quarter being neglected to make the Division more easie, and also for the parts of Inches not set down when you look the Dimensions.

Thirdly, Sometimes Walls have Water Tables, which are two Inches thicker than the forementioned thickneses; in such case measure the Wall at the same thickness 'tis above the Water Table; and instead of setting down the Dimensions of two Inch work, set down but half one of the Dimensions.

For Example, Suppose the Dimensions of a Water Table or piece of two Inch Work be $\begin{matrix} 18. 6 \\ 12. 2 \end{matrix}$ then halve one of the Dimensions, and it will be a Dimension of 4 Inch Work; *that is*, of Work half a Brick thick, and will stand thus,

$$\begin{array}{r} \frac{1}{2} B. \\ 18. 6 \\ 6. 1 \end{array}$$

Fourthly, In Measuring of Chimneys do thus; if the Chimney stand alone, *that is*, not leaning against, or being in a Wall, girt it about below the Mantle, or measure the Back and one Jaum and double it, and take that for the length, and the height of the Room for the breadth; these two Dimensions multiplied together, gives you the Content in the same thickness the Jaums are.

But if the Chimney stand against the Wall of the House which was measured before, then you must girt it round to the Wall, and multiply that into the height of the Room, the Product gives the Content. And here take notice that no deduction is made for the Vacancy betwixt the Hearth and the Mantle.

Mantle-tree, because of the wieths and thickning of the next Hearth above.

Fifthly, For the Chimney Shafts, you must girt them round in the smallest part for the breadth, and take their height for the length; these two multiplied together gives a Content in the same thickness as their breadth; for they are always measured as if they were one solid piece of Brick work, in consideration of the Pargeting and Scaffolding, which is required to bring them up.

Sixthly, In the measuring of Brick Arches we take the breadth of the Arch, and half the breadth and add them together, for the true breadth or measure of the Curve of the Arch, this multiply'd by the length gives the Content in the thickness of the Arch; If there be several Arches together, some Allowance must be made to the Workman, for the Vacuities or Spandrels, which are commonly fill'd up.

Seventhly, Note that all kind of Ornamental Work, as Architrave, Frieze, Cornice, Arches over Doors and Windows, Rubb'd Returns, Rustick Quoins and Mouldings, are generally measur'd by the Foot running Measure.

Eighthly, Peers, Necks, Pilasters, and such like work, is valued at so much a piece.

Ninthly, *Tyleing* is measur'd at so much the square, that is, 10 Foot every way, or 100 square Feet; only *Hyps*, *Valleys* and *Water-Courses*, are measured by the Foot running Measure.

Lastly, Paving with Brick is commonly done by the Square Yard.

Instructions for Measuring of Carpenters Work.

The greatest part of Carpenters work, as Flooring, Roofing, partitioning and Ciel Joys are measured by the square, which Square contains 100 sq. Feet; Or it is such a piece of Work as is just 10 Foot long and 10 foot broad; and in Bills of measurement is generally rated at so much the square.

Windows are usually set down at so much a Light, or so much a piece

Door Cases at so much a piece, either within or without Doors.

Mantletree and Tarsels at so much a piece.

Rayling, Paleing, Lintoling, Guttering, Cornice and Window Board, at so much the foot running Measure.

Stairs at so much the Step, or so much a piece.

Instructions for Measuring of Glaziers Work.

The readiest way of Measuring Glaziers work is with a sliding Rule, by which the Dimensions are taken very exact, even to $\frac{1}{4}$ of an Inch; they commonly agree for their Work by the Foot, whether the Glafs be old or new, Squares or Quarries.

And here note, That if in measuring a Window, there be more Lights than one of the same bigness; as suppose you were to measure an eight Light Window, and each of the upper Pains were 3 foot 2 Inches high, and 2 foot 1 Inch broad, and the lower Pains were 8 foot 6 Inches high, and 4 foot 8 Inches broad, then you need make but two Dimensions of these 8 Pains of Glafs, by setting them down as followeth.

(163)

F. L.

$$\begin{array}{r} 4 : 2 \\ 2 : 1 \end{array} \} 4 \text{ times.}$$

$$\begin{array}{r} 8 : 6 \\ 4 : 8 \end{array} \} 4 \text{ times.}$$

Sarth Windows are Glazed by the Square; *that is*, we tell how many Squares there is in all the Lights, and then reckon what they come to, either at so much the Score, or so much the Dozen.

Instructions for Measuring Joynery, Painting, Plastering and Paving Work.

Joyners, Painters, Plaisterers and Paviers Works are generally agreed for by the Yard; and therefore having cast up your Dimensions, bring their whole Sum into Yards, by dividing the Feet by 9.

Now in taking the Dimensions of Joyners or Painters work, such as *Polletion* and *Bead* work, you must by a Line girt the Bends and Hollows, and so bring it down to the bottom; after which measure the length of your Line by your Rule, setting down this length for one of your Dimensions, then measure the length of the former height, and set down that for a second Dimension.

Note, That Window-Shuts and Doors are measured work and half, because they are work on both sides.

The

The Painting of Windows are generally agreed for at so much the Light, and Casements at so much a piece.

Plastering is also measur'd by the square Yard, and when a Sommer or Girder lies below the Ceiling it is commonly deducted, where the Workman finds Materials.

The Measuring of Masons Work.

Masons Work, whether it be superficial or solid, is measured by the Foot; if you measure the Workmanship only of Stone Steps, then you must measure both ends of the Step, as well as its length.

Of Deductions in general.

First, In Brick work, you are to deduct for all Doors, Windows, and other Vacancies that are measured in.

In Carpenters work, particularly Flooring, you must make deductions for the Well-hole for the Stairs, also for the Chimney Hearths, and the like.

Also in Partitioning, deduction must be made for all Doors and Windows that are measured in.

In Roofing we make no deductions for Window Shafts or Sky Lights, because they are more trouble to the Workman, than the Stuff is worth that would cover them.

In Plasterers work, where the Workman finds Materials, there deduction is to be made for all Vacancies which are not Plastered and yet are measured in.

*Instructions for Measuring, Digging of Cellars,
Vaults, Draines, or the like.*

Digging is generally agreed for by so much the Yard solid Measure; and therefore, for finding the quantity of this kind of work three Dimensions must be taken, which are set down, and cast up as followeth.

F. I.

26. 8

8. 6

4. 9

26. 8

8. 6

208.

5. 4

13. 4

226. 8

4. 9

904.

2. 8

169. 5

1076. 7 Solid Content in F.

Y. F. I.

27) 1076 (39. 23. 7. Solid Content.

266

23

M

And

And after this method are all solid Dimensions cast up.

And here Note, that a Solid Yard of Earth, is accounted one Load.

Having thus given you a particular account of measuring most sorts of Works, I shall in the next place give a short Specimen of drawing out a Bill of Measurement.

*Bricklayers Work done for A. B. at his House
in C. D. by E. F.*

For 50 Rod 68 Foot of Brick work, reduc'd to Brick and half, at 5 l. the Rod.	}	l. s. d. 251 : 05 : 00
---	---	---------------------------

For 17 Square 75 Foot of plain Tyleing at the Square, at 2 l. the Square.	}	35 : 10 : 00
---	---	--------------

For 250 Foot of Rub'd Return, at 3 s. the Foot.	}	37 : 10 : 00
--	---	--------------

For 75 Foot of straight Arch at 2 s. the Foot.	}	7 : 10 : 00
---	---	-------------

For 36 Foot of Jack Arch, at 2 s. 3 d. the Foot.	}	4 : 01 : 00
---	---	-------------

For 1679 Foot of Paveing with 10 Inch Tyle, at 7 d. the Foot.	}	48 : 19 : 05
--	---	--------------

The Sum. 384 : 15 : 05

Measured, July 27, 1699.
by me E. F.

After this manner, changing what is to be changed, are Bills of Measurement for all other sort of Works formed and worded.

Directions

Directions for Measuring Board and Timber.

Board and Plank is commonly Measured by the Foot square; and therefore for finding the Content of any piece of Board, only two Dimensions are requisite to be taken, which are set down and cast up as in the preceeding Examples has been taught.

Dimensions:

$$\begin{array}{r} 24. \quad 8 \\ 1. \quad 11 \end{array} \left. \vphantom{\begin{array}{r} 24. \quad 8 \\ 1. \quad 11 \end{array}} \right\} 47.3.4$$

$$\begin{array}{r} 13. \quad 10 \\ 8. \quad 11 \end{array} \left. \vphantom{\begin{array}{r} 13. \quad 10 \\ 8. \quad 11 \end{array}} \right\} 12.8.2$$

If a piece of Board or Plank be broader at one end than the other, add both ends together, and take half that Sum, or else Measure it in the middle for the true breadth, and Multiply it by the length for the Content.

But suppose a piece of Board or Plank be 3 Inches broad, and you would know how much in length will make a foot square.

Then because a square Foot contains 144 square Inches; therefore if I divide 144 by 3, the Quote 48 is the Inches in length, which of that breadth will make a Foot square.

Hence if you divide 144 by any given breadth, you get the length that will make one Foot square to that breadth, the reason of which is evident; for if you Multiply the Divisor and Quote, that is the length and breadth, the Product will make 144, the square Inches in a square Foot.

M 2

And

And after this manner is the Table upon the common two Foot Rule made, call'd the Table of Board Measure, which shows upon any breadth how much in length will make a Foot square.

<i>A Table of Board Measure.</i>								
1	2	3	4	5	6	7	8	<i>Inch.</i>
12	6	4	3	2	2	1	1	<i>Foot.</i>
0	0	0	0	4.8	0	8 3/4	6	<i>Inch.</i>

By this Table it appears that if a piece of Board be 1 Inch broad, 'twill require 12 Foot in length to make a Foot square; if 2 Inches broad, 6 Foot in length will make a Foot square; if 8 Inches broad, 1 Foot 6 Inches will make a Foot square.

To Measure Board by Gunter's Line.

Exam. 1. Suppose a piece of Plank be 196 Inches long, and 22 Inches broad, What's the Content?

Extend the Compasses from 1 to 196 the length, the same Extent laid the same way, will reach from 22 the breadth to 43 1/2, the Content in superficial Inches:

But if you will have the Content in Feet, then Extend the Compasses from 144 to 196, the same Extent laid the same way, will reach from 22 to 33 Foot near, which is the Content of the Plank.

If the Dimensions be given in Feet and Inches, make an allowance for the odd Inches in parts of a Foot, by taking so many Feet and a-half, or so many Feet and a Quarter, &c.

Exam.

Exam. 2. Suppose a piece of Board be 16 Foot 9 Inches long, and 2 Foot 3 Inches broad, How many square Feet is contained therein?

Extend the Compasses from 1 to 2 Foot $\frac{3}{4}$ the breadth, the same Extent will reach from 16 Foot $\frac{1}{2}$ to 37 Foot $\frac{1}{2}$ the Content near.

Exam. 3. A piece of Board is 17 Inches broad, and 33 Foot long, How many square Feet is contained therein?

Because the Breadth is given in Inches, and the Length in Feet, extend the Compasses from 12 to 17 the Breadth, the same Extent laid the same way, will reach from 34 Foot the length, to 48 Foot $\frac{1}{2}$ the superficial Content.

Exam. 4. Suppose a piece of Board be 10 Inches broad, how much in length will make a Foot square.

In this case you must extend your Compasses from 10 the breadth to 12, the same Extent being laid the same way, will reach from 12 to 14 Inches $\frac{2}{3}$, the length that will make one Foot square.

The Measuring of Timber.

Timber is measured by the Foot solid Measure, and therefore for Measuring of squar'd Timber three Dimensions must be taken, *that is*, Length, Breadth, and Depth, by which Dimensions the Content in solid Feet of any piece of Timber may easily be found.

And here note, That whether Timber be round or square, if it be but as big at one end as it is at the other, it may easily and exactly be measured; but if it be bigger at one end than the other, then whether it be round or square, there is trouble in Measuring it exactly by Arithmetick.

Now before I proceed any farther, it will be necessary to shew you some Errors that are frequently committed in Measuring Timber.

Errors in Timber Measure.

First, When a piece of Timber is not truly square, but broader than 'tis thick, 'tis common with many Artificers in such a case to add the breadth and thickness together, and take half the Sum for the true side of the square, which is apparently false.

For Suppose a piece of Timber be 13 Inches broad and 3 Inches thick, then say they, the true side is 8 Inches, for 'tis half of 13 and 3 added together, which is manifestly false, for 8 times 8 is 64, and 3 times 13 is but 39; so that by this method the Area of the Base is 25 Inches more than the truth, which Error multiplied by the length will Encrease it considerably; so that Timber Measur'd this way makes it more than really it is.

There is also a second Error, which is more frequent than the former, committed in the Measuring round Timber, and that is this.

They take $\frac{1}{4}$ of the Girt of a piece of round Timber for the side of a Square equal to such a Girt, which that 'tis false is very apparent.

Let the Girt of a piece of Timber be 70 Inches, What's the side of a Square equal thereto? Answer this way 17 Inches $\frac{1}{2}$, But in truth is very near 20 Inches, for the square Root of the Area of a Circle, whose Circumference is 70 Inches, is 19 Inches $\frac{1}{2}$. Now 20 times 20 is 400, and 17 $\frac{1}{2}$ by 17 $\frac{1}{2}$ it gives 306 $\frac{1}{4}$, which multiplied by the Length, will give a Content much less than the truth; so that Timber measur'd this way, makes it much less than it should be.

There

There is yet a third Error, in measuring both round and square Timber, that is bigger at one end than the other, and that is girting it in the Middle, which Custom is also very Erroneous, as we shall examin in the following Example.

Suppose a piece of Timber be 26 Inches square at the greater end, and 18 Inches square at the lesser end, and 36 Foot long, What's the solid Content.

By the Common or False way.

26 *side great end.*

18 *side little end.*

44 *Sum of the sides.*

22 *half sum or side measur'd on the mid.*

22

44

44

484 *A of the mean Square.*

432 *length in I.*

968

1452

1936

209088 *Solid. in Inches.*

(172)

By the True way.

$$\begin{array}{r}
 26 \\
 26 \\
 \hline
 156 \\
 52 - \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 18 \\
 18 \\
 \hline
 144 \\
 18 \\
 \hline
 \end{array}$$

676 A. great base. 324 A. little base.
 324 A. little base.

$$\begin{array}{r}
 2704 \\
 1352 \\
 2028 \\
 \hline
 \text{Root.} \\
) 219024 \quad (468 \\
 \hline
 676 \text{ A. great Base.} \\
 86) 590 \quad 324 \text{ A. little Base.} \\
 \hline
 928) 7424 \quad 1468 \\
 \hline
 144 \text{ a third of the length.}
 \end{array}$$

$$\begin{array}{r}
 5872 \\
 5872 \\
 1468 \\
 \hline
 \end{array}$$

211392 True solid Content in l.
 209088 False solid Content in l,

02304 Difference.

Hence you see the common method gives a Content too little, and therefore if it happens to unite with the former of the two preceding Examples, then it is not so great, for that being too much and this too little, do in some measure mitigate the Error. But

But in case the Peice be round, and you girt it in the middle ; a double Error meets and makes the Content considerably less than it should be ; for the same Error happens in girting a round Tree in the middle, as in measuring the side of a square Tree in the middle.

Having thus given you an account of these Errors, I shall in the next place propose some Examples, and then give you a method that will in great measure remedy the two last of the preceding Errors, and therefore serve very well for practice.

Ex. 1. Suppose a piece of Timber is 2 Foot 3 Inches square, and 20 Foot long, What's its Content ?

$$\begin{array}{r}
 2.3 \} \text{ side} \\
 2.3 \} \\
 \hline
 4:6 \\
 6:9 \\
 \hline
 5:0:9 \\
 20 \text{ length.} \\
 \hline
 101:3:0 \text{ the solid Content.}
 \end{array}$$

Exam. 2. Suppose a piece of Timber be 1-Foot 2 Inches thick, 3 Foot 4 Inches broad, and 12 Foot 6 Inches long, What's the Content ?

$$\begin{array}{r}
 1:2 \text{ thickness.} \\
 3:4 \text{ breadth.} \\
 \hline
 3:6 \\
 4:8 \\
 \hline
 3:10:8 \\
 12:6:0 \text{ length.} \\
 \hline
 46:8 \\
 1:11:4 \\
 \hline
 47:7:4 \text{ Solid Content.}
 \end{array}$$

*(Is this Rule of Multy ply,
by foot & parts page
156:.)*

Next

Next for Tapering Timber, whether it be square or round, instead of taking its Dimension in the Middle, Measure or Girt it at something more than $\frac{1}{4}$ of the length from the greater end, and instead of taking $\frac{1}{4}$ of the Girt for the side of the square equal, take $\frac{3}{4}$ of it; These ways though not exactly true, yet will in great Measure correct the former, and serve very well for ordinary Business, or where great exactness is not required.

Exam. 3. Suppose a piece of Timber, at $\frac{3}{4}$ from the greater end Measures 1 Foot 5 Inches square, and suppose it be 23 Foot long, What's the Content.

F. I.

1. 5 } *side*
1. 5 }

1. 5

5

2. 1

2. 0. 1

23 *length.*

46. 1. 11 *Solid Content.*

Exam. 4. Suppose the Girt of a Tapering Tree, at $\frac{3}{4}$ from the greater end, be 63 Inches, and its length 25 Foot, What's the Content?

Here instead of taking one Quarter of 63, which is 15 and $\frac{3}{4}$ I take $\frac{3}{4}$, which is 18 Inches, or 1 Foot 6 Inches.

Feet

(175)

F. 1.

1. 6

1. 6

—

1. 6

6

3

—

2. 3

25. length.

—

50

6. 3

—

56. 3 Solid Content.

—

Exam. 5. A piece of Timber is 16 Inches one way, and 4 Inches the other, How much in length will make a Foot solid.

16

4

—

64 Inches.

—

64) 1728 (27 I

—

448

—

So that 2 Foot 3 Inches in length will make a Foot solid.

From this Example 'tis evident, when a piece of Timber is either square or thicker one way than the other, How much in length will make a Foot solid; For if you multiply the side of the square by it self, or the thickness and breadth together, and by this Product divide 1728 the solid Inches in one solid Foot, the Quote will give you the length that will make one solid Foot.

The

The reason of which is very plain; for the breadth multiplied into the thickness is the Divisor, and that multiplied by the Quote, which is the length, will make 1728, the Inches in one solid Foot.

And thus is the Table upon the common two Foot Rules made, call'd the Table of Timber Measure, which shows how much in length of any exact Square piece will make a Foot solid.

A Table of Timber Measure.

<i>In. Sq.</i>	1	2	3	4	5	6	7	8	9	10
<i>Feet.</i>	144	36	16	9	5	4	2	2	1	1
<i>In. P.</i>	0	0	0	0	$9\frac{1}{4}$	0	$11\frac{1}{4}$	2	$9\frac{1}{4}$	$5\frac{3}{4}$

The use of this Table is thus, if a piece of Timber be 3 Inches square, it shews you that 16 Foot in length will make a Foot solid; If it be 7 Inches then you will find that 2 Foot 11 Inches and $\frac{1}{4}$ of an Inch in length, will make a Foot solid.

To Measure Timber by Gunter's Line.

Exam. 1. Suppose a piece of Timber is 10 Inches square, and 24 Foot long, what's the Content in Feet.

The Extent upon the Line of Numbers from 12 to 10 the Inches square, being twice repeated the same way from 24 Foot the length, gives 16 Foot $\frac{1}{2}$ *prope* the Content.

Exam.

Exam. 2 A piece of Timber is 8 Inches one way, 12 Inches the other, and 19 Foot long, what's the Content.

Reduce the piece to a Square thus, take the middle betwixt 8 and 12 it gives 10 Inches $\frac{1}{2}$ near, for the side of the Square.

Then the Extent from 12 Inches to 10 Inches $\frac{1}{2}$, being twice repeated the same way from 19 the length, will reach to 13 Foot $\frac{1}{4}$ near, the Content. required.

Or thus, without bringing it to a square. The Extent from 12 to 8 Inches the Depth, laid the same way from 12 the breadth, will reach to $8\frac{1}{4}$ a fourth Number: Then the Extent from $8\frac{1}{4}$ to 12, shall reach the same way from 19 the length to 13 Foot $\frac{1}{4}$, the same Content as before.

Exam. 4. Suppose the Girt of a piece of Timber be 56 Inches and 27 Foot long, what's the Content.

Having taken $\frac{1}{4}$ of 56 the Girt, which is 16 Inches for the side of the square equal, proceed as in the first Example; that is, extend the Compasses from 12 to 16, this being twice repeated from 27 the length, will reach to 48 Foot the Content.

Exam. 4. Suppose the side of a square piece of Timber be 6 Inches square, How much in length will make a Foot solid.

The Extent from the side of the square to 12 Inches, laid twice the same way from 1, shall reach to the Feet and parts of a Foot, that in length will make a Foot solid, in our Example 'twill reach to just 4 Foot.

If the side of the square be more than 12 Inches, suppose 14, then the Extent of the side of the square to 12, being twice repeated the same way from 12, shall reach to the Inches and parts of an Inch, that in length will make a Foot solid;

in this Example 'twill reach a little beyond 10, so that very near 10 Inches in length will make a Foot Solid.

If your Timber be Tapering, then whether it be square or round, take the Dimensions of it at $\frac{1}{2}$ from the greater end, as before has been directed.

Of the Transmutation of Figures.

To reduce any superficial Figure to a Square.

Rule. Extract the square Root of the Area of any Superficial Figure, it gives the side of a Square equal thereto.

Exam. 1. Suppose the Area of a Circle be 2346, what's the side of a Square equal thereto. *Ans.* 48 near.

Exam. 2. Suppose the Area of a Square be given, and 'tis required to find the side of another square 8 times as big.

Rule. Multiply the given Area by the number of times you would have it bigger, the square Root of the Product is the side of the Square required: If you would have it but half or $\frac{1}{2}$, or any other part of a Time so big, then Extract the square Root of a Half or Quarter, &c. of the proposed Area, it gives the side of a Square required.

To reduce any Superficial Figure to Rectangle or long Square, that shall have any assigned length or Breadth.

Rule. Divide the given Area by the proposed length or breadth, the Quote is the length breadth required.

Exam.

Exam. 3. Suppose the Area of a Triangle be 366 Foot, what's the breadth of a Rectangle equal thereto, whose length is 61 Foot. *Ans.* 6 Foot.

To reduce any superficial Figure to a Triangle, whose Base or Perpendicular shall be any assigned Dimension.

Rule. Divide the proposed Area by half the given Quantity, the Quote is the Perpendicular or Base required.

Exam. 4. Suppose the Area of any Figure be 864 Foot, what is the Base of a Triangle equal thereto, whose Perpendicular is 36 Foot. *Ans.* 48.

To reduce any Superficial Figure to a Circle, *that is*, to find the Diameter of a Circle, equal to any given Superficies.

This is no more than what was shown in page 120; that is, having the Area of a Circle to find its Diameter.

And therefore if you multiply any given Area by 1.27324, and extract the square Root of the Product, it will be the Diameter of the Circle required.

To reduce any Superficial Figure to an Ellipsis, that shall have any assigned breadth or length.

Rule. What Figure soever the given Area is, take it for the Area of a Circle, to which find the square of the Diameter, by the Method shown in page 120. then divide the said square by the given length or breadth of the Ellipsis, the Quote will be the breadth or length of the Ellipsis required.

Suppose the Area of a Circle be 6432, what's the length of an Ellipsis equal thereto, whose Conjugate Diameter or Breadth is 46 Foot. *Ans.* 89 Foot near.

To

To find the side of a Cube, equal to the solid Content of any Body.

Rule. *Extract the Cube Root of the given Solid Body, and it gives the side of a Cube equal thereto.*

Suppose the solidity of a Parallelepipedon be 32768 Foot, what's the side of a Cube equal thereto? *Ans.* 32.

If you would find the side of a Cube double, treble, &c. or a half, a third, &c. 'tis but doubling, trebling, &c. or halving, thirling, &c. the proposed Area, and then Extracting the square Root of such Product or Quote, and it gives the side of the Cube required.

To find the Diameter of a Sphere equal to any given Solidity.

This is no more than what was effected in page 153, where the solidity of a Sphere was given to find its Diameter; the difference is only this, whatever the given solidity is, take it for the solidity of a Sphere, and find a Diameter to it, as is taught in the forementioned Page.

Lastly, ' If you would reduce any Superficial figure to any other Superficial right lin'd figure, 'tis done both for Regular and Irregular figures universally by *Theo.* 28. to the use of which I refer you for an Example.

' The like also, for Encreasing or Augmenting of Solids in a given Ratio, is shewn in the Use *Theo.* 45. to which I also refer you.

How to make a Vessel of any Form, to Hold any Quantity of Liquor, Corn, &c.

Before I pass to this necessary part of Geometry, it will be requisite to inform you that,

282 Cubick Inches is 1 Ale or Beer Gallon.
 272 Cubick Inches is 1 Corn or Dry Gallon.
 231 Cubick Inches is 1 Wine Gallon.

And therefore,

Cub. In.			
2538	} makes	1 Firkin	} Beer.
5076		1 Kilderkin	
10152		1 Barrel	
2256		1 Firkin	} Ale.
4512		1 Kilderkin	
9024		1 Barrel	
4158		1 Rundlet	} Wine.
14553		1 Hogshead	
19404		1 Tercion	
29106		1 Pipe	
58212		1 Tunn	

Dry Measure.

Cub. In.		
544	} make	1 Peck.
2176		1 Buch. Land Measure.
2720		1 Buch. Water Measure.
54400		1 Hundred of Lime, or 25 B.
78336		1 Chaldron, or 36 B.

To find the side of a Cube, to hold any Quantity of Liquid or Dry Measure.

Rule. Find the Cube Inches in the Vessel proposed, the Cube Root of which is the side of a Cube equal.

Exam. 1. What's the side of a Cube that will hold Eight Beer Barrels.

N

10152

10152 Inches in 1 Beer Barrel.

8

81216 Cube Inches in 8 Beer Barrels.

the Cube Root of which is 43.4 near: the side of a Cube that will hold Eight Barrels of Beer.

Exam. 2. What's the side of a Cube that will hold one Chaldron of Coals.

2720 Inches in a Buch. Water Measure.

36

26320

8160

97920 Cubick Inches in a Chaldron.

the Cube Root of which is 46.2 near, the side of the Cube that will hold 1 Chaldron.

If you would make a Parallelepipedon or Prism of any Limited Base, whether square or oblong, or of any assigned height to hold any Quantity; Then divide the whole Solidity by the Area of the given Base, or by the height, the Quote is the height or Area of the Base, the square Root of which is the side required.

Example 3. What's the side of a square Base of a Parallelepipedon, whose height is 6 Foot 9 Inches, that will hold 6 Chaldron of Coals.

(183)

78336 Cubick Inches in a Chaldron.
6

470016 Cubick Inches in 6 Chaldron:

F. I.

6. 9

12

81 Inches heigh.

81) 470016 (5802 Area of the Base.

650

216

54

Note. That for the 54 Remaining, I add 4 Tenths to the Quote, so that the Area of the Base is 5802.4 Inches, the square Root of which is 76.1 Inches near the side of the Base.

Exam. 4. I would have a Cestern, the Length of which let be 74 Inches, the Breadth 26 Inches, How High must it be to hold 6 Barrels of Water.

Rule. Find the Cubick Inches in 6 Barrels, then by the Product of 74 Inches into 26, that is, by 1924 Divide the aforesaid Cubick Inches, the Quote is the Height of the said Cestern.

Work thus, and you shall find that 31.7 near will be the height of such a Cistern.

N 2

Exam.

Exam. 4. Of what Diameter must a Cylandrick Vessel be, whose height is 3 Foot, to hold two Barrels of Water?

Rule. Find the Cubick Inches in two Barrels, Divide those Inches by the height of the Cylander, the Quote is the Area of the Base; which being a Circle, find the Diameter as is directed in page 120.

Thus proceed, and you shall find the Diameter of the Cylander to be 26.4 Inches near.

From these few Examples and what preceeds, 'tis evident how to change any kind of Vessel into any other kind; as also to make a Vessel of any shape, to hold any Quantity of Liquor, Corn, &c.

I shall here Incert two Tables of the Proportion betwixt the Weights and Magnitudes of Metals; the first shewing the Proportion of the Weight of each Mettal to one another, when the Bodies or Figures are equal: The second shewing the Proportion of the sides of like Solids, when the weight of the Metals are equal.

First Table.

Tin	_____	1554
Iron	_____	1680
Copper	_____	1890
Silver	_____	2030
Lead	_____	2415
Quick-silver	_____	2850
Gold	_____	3990

The first Table shews you that if any Body of Tin, suppose a Cube weight 1554 Grains, the like Body of Gold of the same Bigness shall weigh 3990 Grains.

The

The Second Table.

Gold	—————	3895
Quick-silver	—————	5433
Lead	—————	6435
Silver	—————	7161
Copper	—————	8222
Iron	—————	9250
Tinn	—————	10000

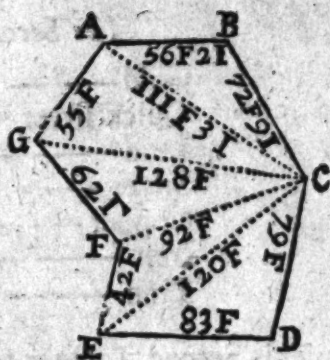
This Second Table is to shew you the Proportion or the Sides when the Weights are equal. As suppose a piece of Gold and a piece of Tin of equal weight, then if two Cubes be made of these two pieces of Metal, the side of the Cube of Tin, shall be to the side of the Cube of Gold, as 3895 to 10000.

*How to take an exact Plot or Draught
of a Court, Yard, Garden, &c.*

THE Practice and Method of performing this, is grounded upon the Problem of making a Triangle with any 3 given Lines, provided any two of them taken together be longer than the third, See *Prob. 1.* with its use.

Supposing therefore the Artist be ready at constructing this Problem, I shall proceed to shew how to effect what I propose.

Let ABCDEFG be a Court or Yard, whose Plan is required; First take a view of it by casting your Eye quite round it, Then upon a piece of Paper delineate the shape of it as near as you can by the hand, It matters not how false or disproportional the Lines are, provided you make the same number in the Draught



as there is really in the Court or Yard, and that you put them nearly in the same situation.

This done, with a Chain divided in half Feet, or else a 10 Foot Rod divided into Feet and Inches; upon a Line strain'd measure the length of every side, *that is*, of the side AB, BC, CD, &c. setting down upon this rough Draught the Dimension or length of each side as you find it by Measuring.

For Example, the side AB is in length 56 F. 2 I. I therefore set down upon the line AB in the rough Draught 56 F. 2 I. In like manner the Line BC measures 72 F. 9 I. this I set down upon the Line BC, and so upon all the rest.

Having thus rounded it, go to some corner, from whence you best may see, and most conveniently measure to all the other Angles; as in this case I go to the Angle C, and from thence with my Rod measure to the Angles A. G. F. E. which Lengths I set down upon the corresponding Lines in your rough Draught: The Dimensions being thus taken, you may by help of the forementioned Problem, draw a fair and true Plan thereof, as follows.

How

How from this Rude Draught to draw a fair one.

Upon the Paper on which you design to draw it make a point as C, to represent the Corner from whence you measured the Diagonals, through this point draw a blind Line at pleasure for your first Diagonal, on which lay off from C to A 111 Foot, 3 Inches, its measure taken from some Duodecimal Scale; for if you actually measure by Feet and common Inches, it must be taken from such a Scale, then take the side A B 56 Foot 2 Inches from the said Scale, and setting one Foot in A, with the other describe an obscure Arch toward C; next take the side C B 72 Foot 9 Inches from the said Scale, and setting one Foot in C, with the other cross the former Ark in B, then draw the Lines C B and A B in Black, Red or what Colour else you please, so have you gotten two of the Boundary Lines of your Draught.

Next, For your second Diagonal C G being 128 Foot, take 128 Foot from your Scale, and setting one Foot in C, with the other make a blind Arch at G, then take 55 Foot the side A G, and setting one Foot in A, with the other cross the former Arch in G, and draw A G, so have you got a third boundary Line.

Again, take 92 Foot the third Diagonal, and setting one Foot in C, with the other strike a blind Arch at F, and with the Distance G F 62 Foot, and one Foot in G cross the said Arch in F and draw G F, so is a fourth Line of your Arch finished.

This done, Take 120 Foot from your Scale, and with one Foot in C make a blind Arch at E; then

take FE 42 Foot, and with one Point in F cross the former Arch in E, and draw FE for the fifth boundray Line of your Draught.

Lastly, take the side ED 83 Foot betwixt your Compasses, and with one Foot in E describe an obscure Arch at D; then take 79 Foot the length of the side CD, and with one Foot in C cross the Arch in the point D; draw the Lines ED and CD, and your Draught is finished.

This method may likewise be used for small Fields or Enclosures.

Another way to do the same.

Sometimes it may happen, that by reason of old Stuff or Rubbish lying upon the Plot or Plan, Diagonals cannot be Measured; in such case the following Method will be very serviceable.

Let the Figure ABCD following be such a Ruinous Foundation, whose Plot is required.

Draw a rough Eye-Draught, and measure the several boundary Lines, whose measures set down upon the corresponding Sides as before; this done, go to the Angle A and measure upon the side AB 6 Foot to *g*, (or as occasion requires more or less) also measure upon the side AD 6 Foot to *e*, (or more or less) from *e* to *g* strain a Line and measure the length thereof, which let be 8 Foot; this set down upon the prick't Line *eg* in your Draught.

Do the like at the Angle B, and set these down also; so have you Dimensions sufficient to plot your Draught as followeth.

To draw a Fair Plot of the same.

Upon a Sheet of clean Paper draw a Line, and lay thereon from A to B 28 Foot, taken from your Scale.

Then

Then take 6 Foot from the Scale, and lay from A to *g*, and from B to *h*; upon the Points A and B describe two prick't Arches at *e* and *k*; then because the Diagonal *ge* is 8 Foot, take 8 from the Scale, and with one point in *g* cross the former Arch in *e*, from A through *e* draw a Line at pleasure, and lay thereon from A to D 27 Foot.



Again, Because the Diagonal *hk* is 9, I take 9 from my Scale, and setting one Foot in *h*, with the other cross the former Arch in *k*; then from B through *k* I draw a Line at pleasure, and lay thereon from B to C 26 Foot for the side BC: This done, from the points D and C draw the Line DC and the Plot is finished, which if right, the Line DC will measure upon the Scale 21 Foot, as it really is.

Note, The prick't Lines *eg*, *hk* must not appear in a fair Draught.

Your Plot being fairly drawn, the Content is found in Feet and Inches by cutting it into Triangles, as is shewn in *Prop. 4.* of the preceding Treatise of Measuring; If the Content of it be required in *Perches*, *Roods* or *Acres*, then divide the Content in Square Feet, by 272.25 the Quote will be square Perch; Or divide the square Feet by 10890 the Quote will be square Roods; Or if you divide by 43560, the Quote will be *Acres*.

THE
Manner of Cutting
 AND
 MEASURING
The Five Regular Platonick Bodies.

LET the Diameter of the Sphere be 2, then the Sides of the 5 Regular Bodies inscrib'd in the same, will be as follows.

Diameter of the Sphere 2.
 Side of the Tetraedron 1.6229.
 Side of the Cube 1.3547.
 Side of the Octaedron 1.4142.
 Side of the Dodecaedron 0.7136.
 Side of the Icosaedron 1.0514.

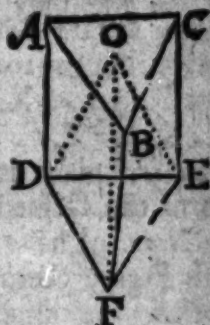
To cut the Tetraedron.

Make a Triangular Prism, whose Base is an Equilateral Triangle, and side of which must be 1.62, &c. and the height of the said Prism $\frac{2}{3}$ of the Diameter of the Sphere, *that is*, $\frac{2}{3}$ of 2 = $\frac{4}{3}$ or $1\frac{1}{3}$. This done find the Center of the Triangle (*Prob. 17*) then let Plains be conceived to pass through the Points DFO, EFO, DEO, *that is*, shave the Wood exact to these Points, and you'll form the Tetraedron required.

To Measure it.

This Body being a Triangular Pyramid, must be measured by the Rule for measuring a Pyramid, viz. by multiplying any one of the Triangular Plains by $\frac{1}{3}$ of the nearest distance betwixt one Plain and the point opposite, the Product is the solid Content.

Its superficial Content is found by multiplying the Area of any one of the Triangular Plains by their Number, which is 4, the Product is the Area of its Surface.

*How to cut the Cube.*

For the cutting of this Figure there needs no direction, only let the side be 1.1547 and plain it exactly square.

To Measure it.

Multiply any one side by it self, and that Product again by it self, this last Product is the solid Content.

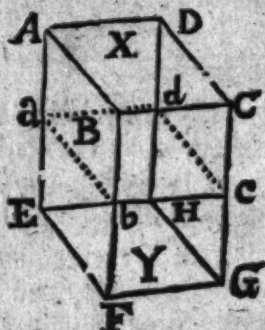
Its Superficial Content is found by multiplying the Area of any one of the Bases by their Number, which is 6, the Product is the Superficial Content.

To cut the Octaedron.

Let there be made a Parallelepipedon ABCDEFGH whose Base is a square, the side of which must be 1.41, &c. and the height the Diameter of the Sphere, which is 2.

Bisect

Bisect the Lines AE , BF , CG , DH in the Points, a , b , c , d , and join those points by the prick'd Lines ab , bc , cd , da ; this done, find the Centers X , Y of both square Bases, then conceive Plains to pass through the Points abX , bcX , cdX , daX ; abY , bcY , cdY , daY ; that is, plain it off to those points, and it will form the Octaedron required.



Note. I have not drawn the Lines of Section in this Figure because it would make too great confusion, the thing being clear without it, for if your Plain passes but through the forementioned Points you cannot help forming the Figure.

To Measure it.

This solid is only two Pyramids abutting upon one common Base, and is therefore measured by multiplying one of its sides by it self, and that Product again by $\frac{1}{3}$ of the Distance of any two Corners, this last Product is the solid Content.

Its Superficial Content is found by Multiplying the Area of any one of the triangular Bases by their Number, which is 8, the Product is the Content of the Surface.

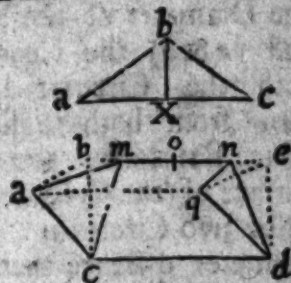
To cut the Dodecaedron.

To cut this Figure whose side is (0.71364) the greatest Segment of the side of a Cube inscrib'd in the said Sphere, when cut in Extream and mean Proportion. Make therefore a Cube fit to inscribe in the same Sphere, the side of which is 1.1547.

Let

Let half the side of this Cube, *viz.* 5773 be cut in Extream and mean Proportion, then form an Isosceles, whose Base is *ac* the side of the Cube; and its Altitude *bx* ($= .71364$) the greater Segment of half the side of the Cube cut as aforesaid.

This done make a Triangular Prism *abcged*; having this Triangle for its Base and the side of the Cube for its Altitude, bisect *be* in *o*, and take *om* and *on* severally equal to *bx*, then let plains pass through the points *acm* and *adn*, *qdn*, that is, plain off the Wood to these points *acm* and *adn*, Make 6 of these Figures, which glew'd, or otherwise fasten'd to the Faces of the Cube; will make the Dodecaedron required.



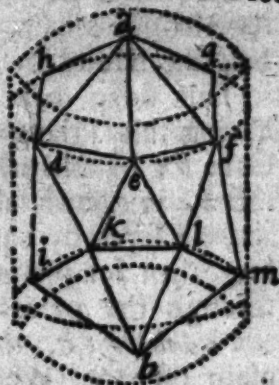
To Measure it.

This Body is made up of 12 equal Pyramids, the Bases of which are the Pentagonal Plains that bound the Figure, and the tops of the Figure the Center of it; find therefore one of these Pentagonal Bases, and multiply it by $\frac{1}{5}$ of its distance from the Center, this Product multiply by 12, it gives the solid Content of the Figure.

Its superficial Content is found by multiplying the Area of one of the Pentagonal Plains by 12.

To cut the Icofaedron.

Make a Cylender, the Radius of whose Base let be (0.8944) the square Root of $\frac{1}{2}$ of the square of the Diameter of the Sphere, and its Altitude (2) equal to the Diameter of the Sphere, find (*a* & *b*) the Centers of both the Circular Bases, then from both these Circular Bases on the curve Surface of the Cylender, describe two Circles at $\frac{1}{2}$ of the Diameter distance therefrom.



Divide any one of them, suppose the upper into 5 equal parts, and likewise the under Parallel after the same manner, only with this difference, that the beginning of the Divisions on this, may be Perpendicular over the middle of the other Divisions, then let Plains pass through the Points *bda*, *dea*, *efa*, *fga*, &c. Also *dik*, *dek*, *kel*, *lef*, *lfm*, &c. *ikb*, *kib*, *lmb*, &c. and you will form the Icofaedron.

To Measure it.

This Figure is made up of 20 equal Pyramids, whose Bases are the Triangular Plains that bound the Figure, and height the half distance betwixt two opposite Faces; find therefore the Area of one of these triangular Plains, and multiply by $\frac{1}{2}$ of the half distance of any of the opposite Plains; the Product multiply by 20, it gives the solid Content of the Icofaedron.

Its Superficial Content is found, by multiplying the Area of any one of the Triangular Bases by 20.

The

The Proportions of the Sphere, and of the Five Regular Bodies inscrib'd in the same. As found in Peter Herigon. Cur. Mat.

Let the Diameter of the Sphere be 2.

The Circum. of the greatest Circle	6.28318
Superficies of the greatest Circle	3.14159
Superficies of the Sphere	12.56637
Solidity of the Sphere	4.1879
The side of the Tetraedron	1.62299
Superficies of the Tetraedron	4.6188
Solidity of the Tetraedron	0.15132
Side of the Cube	1.1547
Superficies of the Cube	8.
Solidity of the Cube	1.5396
Side of the Octaedron	1.41421
Superficies of the Octaedron	6.9282
Solidity of the Octaedron	2.33333
Side of the Dodecaedron	0.71364
Superficies of the Dodecaedron	10.51462
Solidity of the Dodecaedron	2.78518
Side of the Icosaedron	1.05146
Superficies of the Icosaedron	9.57454
Solidity of the Icosaedron	2.53615

If you would cut one of these Bodies out of a Sphere of any other Diameter. Say, as the Diameter of the Sphere 2, is to the side of any one of the Solids inscrib'd in the same, suppose the Cube 1.1547; So is the Diameter of any other Sphere; suppose 8, to 9.2376, the side of the Cube inscrib'd in this latter Sphere.

F I N I S

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